INSIGHT LEARNING TECHNOLOGY, INC.

922 Las Lomas Avenue
Pacific Palisades, CA 90272
www.insightlearningtech.com

Background and Validation Research on Perceptual Learning and Sequencing Technologies

-- *Summary* --

Insight Learning Technology, Inc. offers revolutionary approaches to learning technology based on current research in cognitive science that are applicable to almost any learning domain. Two primary innovation areas drive our technology: *perceptual learning methods* and *optimal sequencing algorithms*. This document provides some scientific background and a sample of research results showing the effectiveness of perceptual learning and sequencing in real learning contexts, with the focus on mathematics and science learning.

I. Purpose

The mission of *Insight Learning Technology, Inc.* is to revolutionize learning in the home, school and workplace, using cognitive science and digital technology. Given the number and diversity of efforts that already exist in learning and learning technology, how can we presume to do this? The answer is that Insight's approach is based on research in cognitive science that is not generally known or applied, including our recently patented sequencing technology and patent pending perceptual learning technology. Our approach allows us to recognize and address crucial but neglected aspects of learning and to make use of unrealized potential inherent in computer-based learning.

In this document we provide very brief background on our two major innovation areas: perceptual learning techniques and optimal sequencing algorithms. We then give an overview of validation research, emphasizing mathematics, that indicates the efficacy of both perceptual learning and sequencing methods.

It should be noted that the particular learning modules discussed here represent specific, but in no way limiting, examples of our learning technology. The modules described illustrate the consistent effectiveness and applicability of perceptual learning and sequencing technology. The technologies are applicable to many learning domains in mathematics and elsewhere. Our technology and basic PLM Shell Program make it comparatively easy to devise new modules for specific purposes (e.g., a set of modules to correspond to chapters or units in a pre-algebra or geometry text). The research data here suggest that future learning modules based on our technologies would be likely improve the efficiency of learning and address dimensions -- such as pattern recognition and fluency -- that are poorly addressed by other methods.

II. Brief Scientific Background

A. Perceptual Learning Techniques and Perceptual Learning Modules

Perceptual learning modules (PLMs) describe a new approach to learning technology with wide applicability (Wise, Kubose, Chang, Russell & Kellman, 2000). PLMs can be applied to mathematics, science, reading and other subjects in primary, secondary or university settings, and many professional and industrial learning domains. They are well-suited for web-based instruction and distance learning.

PLMs draw on research about how people learn and develop expertise as well as on recent advances showing amazing plasticity, specifically driven by perceptual learning, in the neural circuitry of the brain (for a review, see Kellman, 2002). Ordinary classrooms and current educational technology emphasize facts and verbal concept learning, which are important, but these approaches neglect other crucial aspects of human learning: pattern recognition skills and intuitive grasp of key structures (Gibson, 1969; Bereiter & Scardamalia, 1998). Although it is well recognized that experts in many domains, such as mathematics, science, air traffic control, chess, radiology, etc. see key patterns quickly and effortlessly, it has long been believed that such abilities cannot be taught. Fluency and intuitions about structure are said to develop only from years of experience. Yet the absence of fluency and pattern recognition, and the lack of reliable ways to teach them, hamper instruction in mathematics, as well as in other fields. Even students who learn the facts and procedures often show poor fluency, failure to transfer, and lack of recognition of when facts or procedures apply.

Research shows, however, that these aspects of learning can be dramatically accelerated. Doing so requires the interactive learning techniques embodied in PLMs. Work on perceptual learning modules by Kellman and colleagues began at NASA Ames Research Center in the early 1990s and focused on aviation training. This research showed that for abilities such as visual navigation and interpretation of instruments in instrument flight, brief (under 1 hour) use of a PLM allowed novices to surpass 1000-hour civil aviators (Kellman & Kaiser, 1994). Experienced pilots also showed striking improvements in both accuracy and speed in visual navigation, and reduced their time for instrument interpretation by 60%. Subsequently, several PLMs were introduced into the instrument flight curriculum at Embry-Riddle Aeronautical University; these were shown to sharply accelerate students' skills in instrument interpretation and use of complex instrument enroute and approach charts (Kellman, Stratechuk & Hampton, 1999).

More recently, we have applied these ideas in educational domains, especially mathematics, with excellent results. Basic research on the effectiveness of PLMs has been done at the UCLA Human Perception Laboratory, under research grants from the National Science Foundation and the US Dept. of Defense, among others. Most of the PLM mathematics research reviewed below has been carried out at the UCLA Lab and at the Institute for Research in Cognitive Science at the University of Pennsylvania under the NSF-sponsored project "Perceptual Learning in Mathematics and Science" (NSF ROLE Program - REC-0231826, 2002 – 2006; Philip J. Kellman, PI; Christine Massey, Co-PI).

B. Optimal Sequencing Technology

Insight's patented optimal sequencing algorithms make learning more efficient in any domain in which there are a number of particular items, procedures, or concepts to be learned. These proprietary methods utilize scientific principles about human long term memory and learning. They arrange the sequence of learning items dynamically, based on the learner's accuracy and speed on prior learning trials. By doing so, they focus the learner's effort where it is most needed, reduce learning time, and lead to better retention. The combination of sequencing with objective learning criteria and problem retirement comprises an extremely powerful integrated process for optimizing learning efficiency and certifying mastery for all parts of a learning task.

Sequencing has been shown to cut learning time roughly in half for memory items such as the learning of multiplication tables, vocabulary words, etc. Research underway suggests that similar savings will be realized in concept learning and pattern recognition learning, in which concept or pattern types, rather than specific memory items, are sequenced. Our learning technology employs continuous comparisons with learning criteria, providing objective indications about which items have been mastered and which items require further study.

Applications are unlimited. Learning in any area in which certain basic information must be remembered, such as basic math facts (e.g., multiplication tables) can be improved by our item sequencing methods. Combining sequencing algorithms with perceptual learning and structure discovery concepts has equally broad implications for higher-level learning, such as learning to classify math and science problem types, algebraic transformations, mapping across representational formats (e.g., graphs, equations, word-problems), biological kinds, chemical families, and so on.

III. Validation Research on Perceptual Learning Modules

In this section, we describe research on perceptual learning modules (PLMs) in 5 areas, spanning different topics in mathematics, and in one case, science learning.

A. *The Algebra-Geometry Connection PLM*

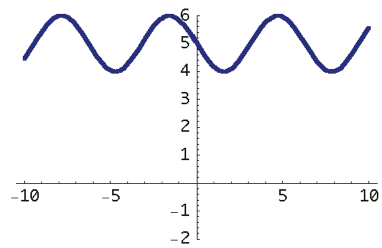
The ability to map across multiple representations is a key component of success in many mathematics and scientific domains. One example is a student's understanding of a function plotted as a graph and given as an equation. One might think of this as the "algebra-geometry connection": understanding how the same mathematical relationships can be expressed symbolically (algebra) or graphically (geometry). Another is a student's extracting of structure and relations from a word problem in order to construct an equation or graph.

These are key concerns in standards for students' secondary school mathematics learning (National Council of Teachers of Mathematics, 1999). For example, NCTM standards for grades 9-12 indicate that instruction should, among other things, help students to: 1) learn patterns and transformations in graphical displays of mathematical functions; 2) learn to connect the written and graphical representations of these functions; and 3) transfer learning from simple, basic functions to more complex ones.

Background research with middle school, high school, and even collegiate students convinced us that intuitive understanding of equations, graphs, and word problems, and the mappings among them, are poorly understood by many students. Despite the inclusion of all of the relevant information in conventional instruction, students' pattern recognition and fluency may be poor. We believed that PLMs would be particularly well-suited to the development of structure understanding and fluency in this domain, among others.

Initial studies involved several hundred students at a highly selective university and focused on functions studied in high school mathematics (such as sine, cosine, exponential and logarithmic functions) and their transformations. The design involved presentation on each trial of a graph, and the learner made a forced choice of which of several possible equations matched the graphed function. Speed and accuracy were continually tracked. Learners worked problems in two function families and were tested afterwards with novel problems in the studied families and also in two novel function families. A number of details of construction of the PLM, as well as a number of experiments that helped to reveal important aspects of active vs. passive trials, relative value of different styles of feedback, and ways of enhancing user motivation, are omitted here.

An example problem is shown below:



y = 5 * Sin (x)

y = 5 - Sin (-x)

y = 5 + Sin (-x)

Figure 1. Sample Classification Problem from the *Algebra-Geometry Connection PLM.* One of the three equations below the graph represents the graphed function.

An interesting aspect of the study was the use of transformations within each function family that could potentially support transfer to other function families. For example, for any function f(x), the graphical consequence of transforming it to f(-x) is to reflect the function across the y axis; a transformation from f(x) to f(nx), for n>1, is a horizontal compression of the function, and so on.

Results. Typical results from the 40 minute algebra-geometry connection are shown below. Figure 2 shows accuracy data and Figure 3 shows speed data. Note that in Figure 3 lower bars indicate better (faster) performance.

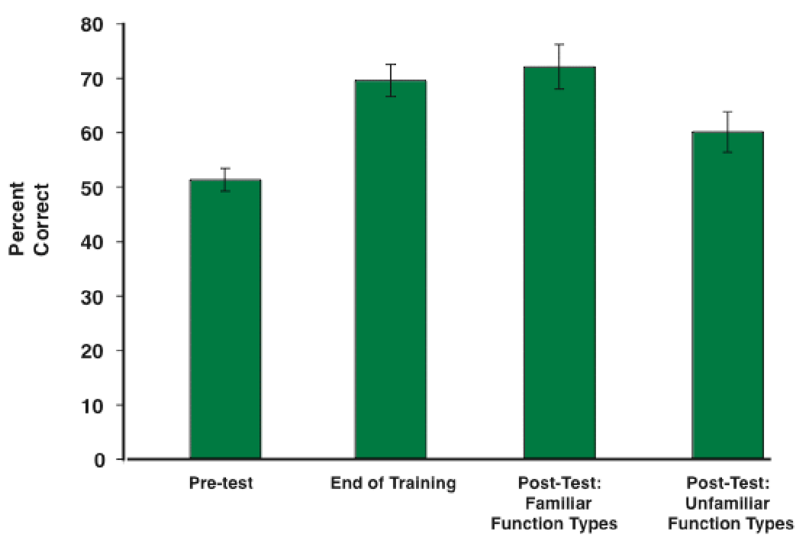


Figure 2. Accuracy Data from Algebra-Geometry Connection PLM. Error bars show ± one standard error of the mean. Pre-test data was acquired from the first two trial blocks (20 trials total) of the learning session. End of training data comprised the last two

trial blocks. Novel (previously unseen) problems were used for the post-test for both familiar and unfamiliar function types.

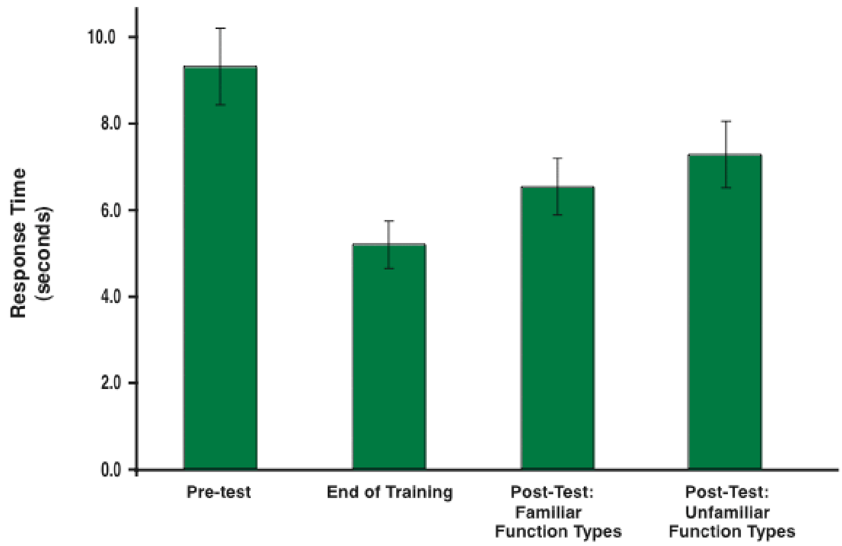


Figure 3. Response Time Data from Algebra-Geometry Connection PLM. Error bars show ± one standard error of the mean. Response time data includes only trials on which answers were correct. Pre-test data was acquired from the first trial block (10 trials total) of the learning session. End of training data comprised the last two trial blocks.

Results of these early PLM studies in mathematics strongly confirmed that even brief use of a PLM could strongly improve mapping across multiple representations and presumably extraction of structure from equations and graphs (Silva & Kellman, 1999). As shown in Figure 2, accuracy acquired during learning transferred to novel instances and, to some extent, to new function families, showing that learners were coming to extract important and general structural invariants. As Figure 3 indicates, learners also experienced strong fluency gains, reducing the time required per trial by about 40% by the end of training.

It is remarkable that a short (40-minute) intervention could substantially boost students' extraction and use of key mathematical structures, despite the fact that the students -- who were undergraduates enrolled in a prominent, highly selective university -- had apparently not previously mastered these skills in conventional courses that they had completed successfully. The results confirm the need to address neglected dimensions of learning as a complement to conventional instructional methods.

A number of variations on the basic experiment have been carried out, investigating several key variables related to optimizing learning in PLMs, including structuring of feedback, use of active classification vs. passive exposure, remote transfer to compound functions, and so on (Kellman, Massey & Silva, in preparation). Results have taught us important aspects of using perceptual learning techniques, most applicable to future applications.

SUMMARY: ALGEBRA-GEOMETRY CONNECTION PLM

- ➔ **LEARNERS SHOWED SUBSTANTIAL GAINS IN ACCURACY AND SPEED FROM A 40-MINUTE INTERVENTION.**
- ➔ **LEARNING GAINS TRANSFERRED TO PREVIOUSLY UNSEEN PROBLEMS, SHOWING ACQUISITION OF ABSTRACT PATTERN STRUCTURE.**
- ➔ **LEARNING TRANSFERRED NOT ONLY TO NOVEL PROBLEMS BUT ALSO SOMEWHAT TO NOVEL FUNCTION FAMILIES, INDICATING INTUITIVE LEARNING OF GENERAL FUNCTION TRANSFORMATIONS.**

B. *Multi-Rep PLM*: Mapping Across Multiple Representations

The concepts of extracting structure from mathematical representations and mapping between representations apply at many levels of the mathematics curriculum. Analogous to the *Algebra-Geometry Connection PLM*, we developed the *Multi-Rep PLM* to help middle and high school students develop pattern recognition and structure mapping with representations of linear functions, in graphs, equations, and word problems.

In the Multi-Rep PLM students had two 40-minute sessions consisting of many short trials. As is true in many of our PLMs, rather than having students solve problems, we presented them on these trials with short, interactive classification tasks which facilitated fluent extraction of important features and patterns. On a typical trial, a learner was shown a "target" display and made a forced-choice classification of which of three response displays showed the same function in a different representation. Students using the PLM mapped across representations involving graphs, equations, and word problems, all of which could appear as targets or response items on different trials. A sample problem is shown in Figure 4. Speed and accuracy were tracked continuously. We omit here a number of details regarding arrangement of different sorts of trials, learning criteria, feedback to ensure learning and sustain motivation, etc.

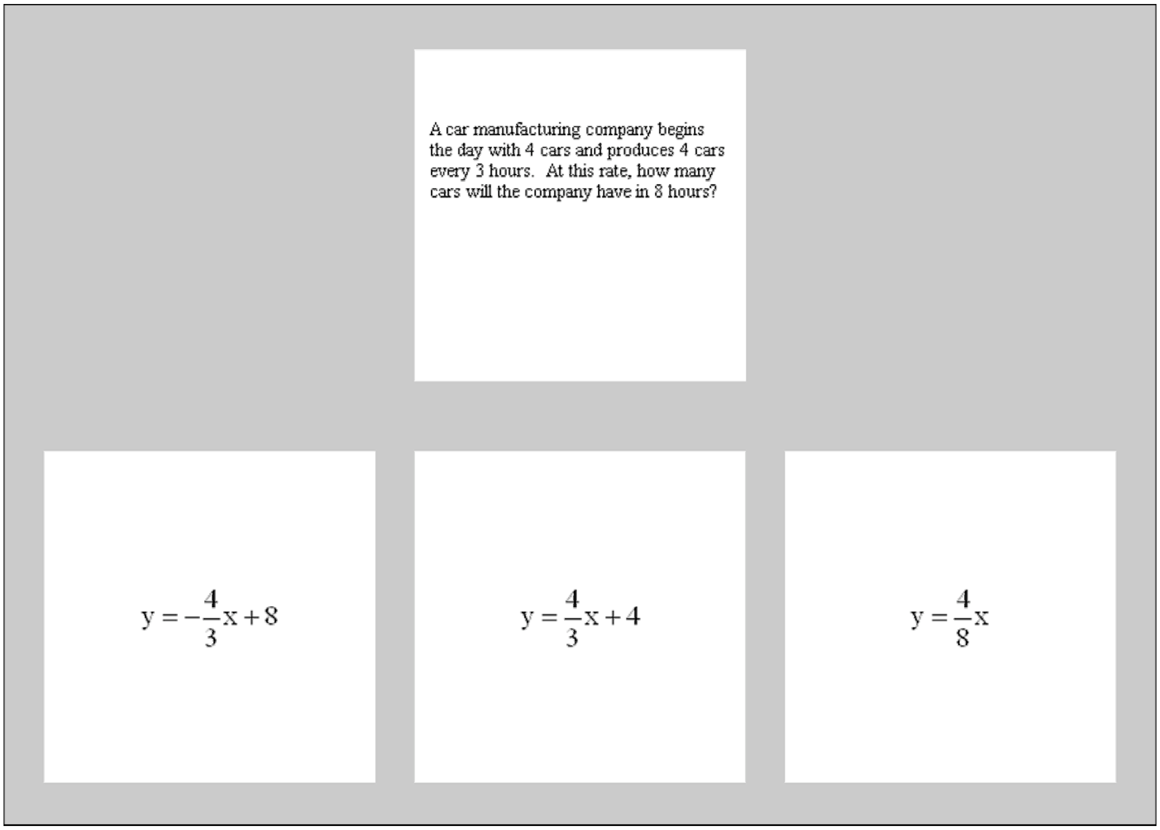


Figure 4. Sample Classification Problem from Multi-Rep PLM. One of the three equations below the word problem has a corresponding structure. Trials in the learning phase included all possible combinations of graphs, equations, and word problems appearing as targets and response options.

We studied different versions of this PLM with more than 60 students at a diverse, philanthropic private school in Los Angeles. To give some idea the benefits of PLMs in these studies, we show data below for two groups, a group of 12th graders and a group of 9th and 10th graders. The younger sample included a control condition and a delayed post-test.

The control group was assigned randomly from the same classes and spent an amount of time on task equal to the PLM group. Their task, however, was to construct graphs and equations from a given graph, equation, or word problem. This was also the task tested for both groups in the post-test. This is not an ideal control group for two reasons. First, we would expect some perceptual learning and structure mapping to be going on in the control task; however, we would expect it to be less systematic than in the PLM group. Second, because the control group's task matched exactly the post-test task, the design is actually somewhat biased against the PLM group, who did not do this "translation" task in the learning phase. That is, the control group directly practiced the same task that was used as the post-test task. The PLM group practiced a different task, so for them the post-test task was a transfer task. In this sense, the comparison constituted a strong test of the idea that the PLM format would improve the students' abilities to see relevant structures and would facilitate transfer to the active construction task (e.g., generating the correct equation from a word problem).

Results. Figure 5 shows data from 12th grade students. A preliminary observation: The data clearly suggest the need for PLMs or similar interventions. Linear functions are emphasized in the curriculum in Pre-Algebra and much of Algebra I. They appear in a wide variety of middle and high school science courses. Yet our pre-test data show that, in general, students' abilities to make basic mappings from equations to graphs and word problems is very poor through high school.

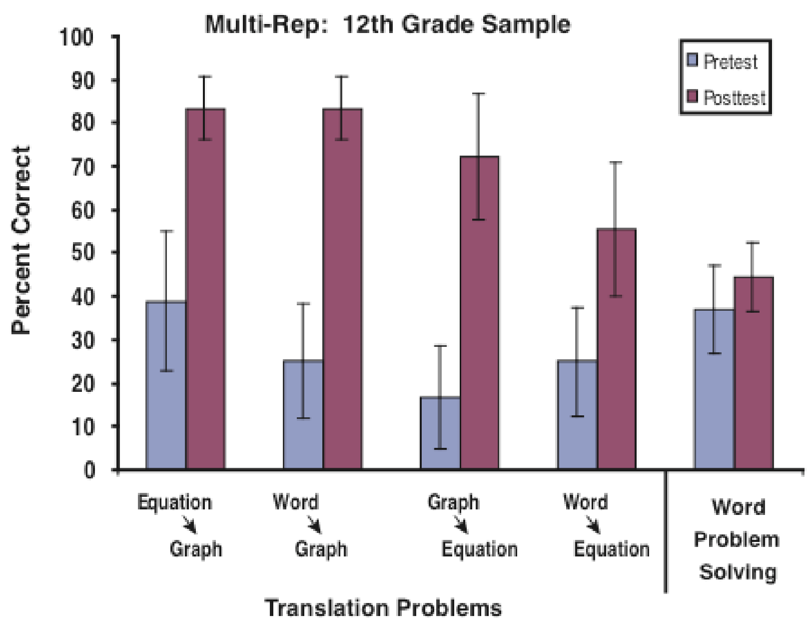


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Figure 5. Accuracy Data from Multi-Rep PLM with 12th Graders. Error bars indicate ± one standard error of the mean. Translation problems involved presentation of an item (word problem, graph, or equation) for which the student had to construct the appropriate equation or graph.

Use of the *MultiRep PLM* for two 40-minute sessions produced dramatic gains in important mathematics skills in the 12th grade sample. Prior to the PLM, students in the sample on average could produce the correct equation from a graph under 20% of the time, or could produce an appropriate graph from a word problem under 30% of the time. After PLM exposure, these students as a group averaged 75-85% correct on these mappings. To map across these representations not only shows an understanding of the correspondences between representations but also of the crucial structural features of each representation, including extraction of structure from word problems, a notoriously difficult task in the educational system. Solving of word problems improved only marginally, but the effect is suggestive. A complexity we are studying is the fact that many students (before PLM exposure) do not solve word problems by generating equations, but do so by using heuristics and shortcuts. With improved structure mapping ability from the PLM, students may begin to reorganize how they solve problems, but this process may be ongoing. (See data from younger sample below.)

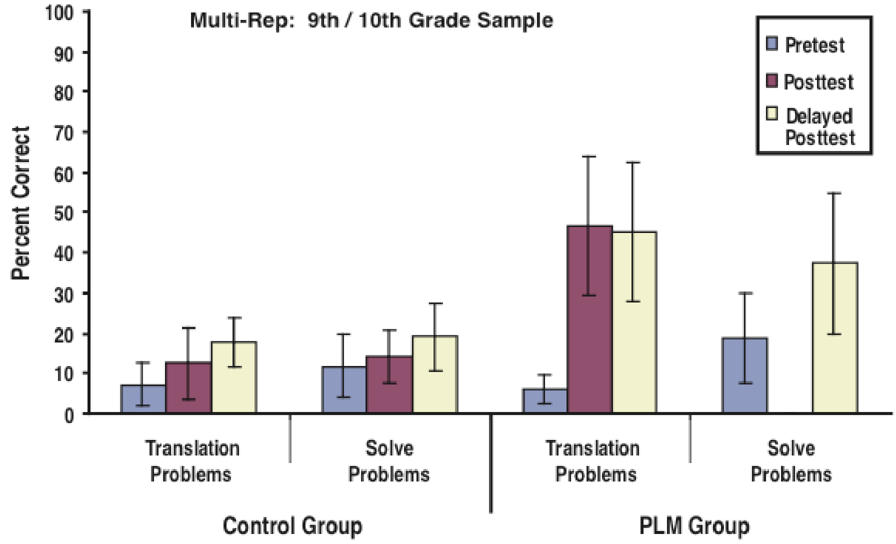


Figure 6. Accuracy Data from Multi-Rep PLM with 9th and 10th Graders. Error bars indicate ± one standard error of the mean. "Translation" problems involve presentation of an item (word problem, graph, or equation) for which the student must construct the appropriate equation or graph. "Solve" problems involved the solving of word problems. The Control Group practiced the post-test translation task during their learning sessions; the PLM did not do any active construction during the learning sessions.

Figure 6 shows the data for the 9^{th} and 10^{th} graders. Despite clear emphasis in standard courses on linear functions, their graphs, equations, and related word problems, the 9th and 10th graders in our sample showed extremely low levels of performance on pre-tests, both for generating function isomorphs across representations and for solving word problems. As in every sample we have tested, PLM usage dramatically raised the levels of accuracy in students' ability to construct a graph or equation, given a word problem, graph, or equation. (These data are combined here as "translation problems.") The PLM students went from around 5% accuracy levels to nearly 50%. The control group showed very modest improvement, far less than the PLM group.

A delayed post-test was administered two weeks later. Results showed that gains in translation problems were fully preserved in the PLM group. There was also a surprising jump in the ability to solve word problems shown in the delayed post-test. Although performance in the PLM group actually appeared to drop in the immediate post-test, it rose substantially after a two-week delay. The pattern is consistent with the idea that PLM training provoked a reorganization of students' approach to problem solving (i.e., they moved away from heuristic methods to systematic methods, such as generating an equation). Further work will be needed to confirm this fascinating idea, as due to subject loss, only a very small sample (n=4) actually completed the delayed post-test.

SUMMARY: *MULTI-REP PLM*

- ➔ **LEARNERS WHO PARTICIPATED IN TWO 40-MINUTE PLM SESSIONS IMPROVED DRAMATICALLY IN THE ABILITY TO GENERATE APPROPRIATE GRAPHS AND EQUATIONS FROM GIVEN WORD PROBLEMS, EQUATIONS OR GRAPHS.**
- ➔ **THESE GAINS FROM THE PLM WERE NOT SHOWN BY A CONTROL GROUP GIVEN DIRECT PRACTICE ON THE POST-TEST TASK.**
- ➔ **IMPROVEMENTS IN STRUCTURE MAPPING WERE LASTING, AS SHOWN BY FULL PERSISTENCE OF LEARNING GAINS ON A DELAYED POST-TEST.**
- ➔ **DELAYED POST-TESTING ALSO SUGGESTED LATENT GAINS IN SOLVING OF WORD PROBLEMS BY THE PLM GROUP.**

C. *Algebraic Transformations PLM*

Students show characteristic problems in learning algebra. After some coursework, they may be able to state the goal and method for solving an equation. Students know they should "get x alone on one side," and "do the same operation to both sides of the equation." Yet this explicit knowledge does not suffice to be good at algebra. Understanding the rules of algebra is crucial, but students must also be able to see structure in expressions and equations, and relations among them. Ultimately, they need to do some visualization of what things will look like if certain transformations are applied. These abilities are very difficult to address systematically in conventional instruction.

We developed our *Algebraic Transformations PLM* in order to apply perceptual learning methods to improve students' pattern processing and fluency in algebra. We developed a classification task in which participants viewed a target equation or expression and made speeded judgments about which one of a set of possible choices represented an equivalent equation or expression, produced by a valid algebraic transformation.

A key goal of this PLM was to contrast the declarative knowledge components (facts and concepts that can be verbalized) with the idea of "seeing" in algebra. Classroom instruction and homework address the former, but are there unaddressed aspects of learning that impede students' progress? Preliminary work convinced us that most students, even in lower-track mathematics classes, understand the rules of algebra (e.g., the idea of solving equations by doing the same operation to both sides). A more difficult challenge is to get students to see the structure of expressions and equations, and relations among them, in order to use transformations fluently. Conventional instructional methods do not directly produce the fluency and pattern recognition that are crucial to being good at algebra. We asked: Can PLMs address these dimensions of learning?

In the *Algebraic Transformations PLM*, we did not ask students to solve problems. Instead we devised a classification task that exercised the extraction of structure and the seeing of transformations. On each trial, an equation appeared, and the student had to choose which one of several options below was a legal transformation. An example is shown in Figure 7.

$$(-p)(6)(y)(4+y) = 6z$$

A	$(-p)(6)(y) = (6z)(4+y)$
B	$(-p)(6)(y) = (6z+p)(6)$
C	$(-p)(6)(y) = \frac{6z}{(4+y)}$
D	$(-p)(6)(y) = 6z - 4 + y$

Figure 7. Sample Classification Problem from the *Algebraic Transformations PLM*. Only one of the four equations below the target represents a valid algebraic transformation of the target equation. The learner's task on each trial is to recognize and select the valid transformation.

In addition to testing whether practice in the PLM improved accuracy and fluency in recognizing transformations, we also examined whether students would be able to transfer learning to solving algebraic equations.

This study was carried out with 42 8^{th} and 9^{th} grade students at mid-year in an Algebra 1 course. Students participated in two 35-minute learning sessions using the *Algebraic Transformations PLM*. On each trial, they were shown a target equation and were asked to select which of four choices could be correctly derived by performing an algebraic transformation on the target. Students were given feedback after each trial indicating whether or not they had chosen the correct answer. Incorrect answers were followed by an interactive feedback screen in which students were shown the target and the correct choice and were then asked to choose from a set of descriptions the one that correctly described the transformation that related the target to the correct choice (e.g., “divide by (4 + y)”).

Results. The task that formed the core of the PLM -- matching an equation to a valid transform -- is directly useful to development of pattern recognition and skill in algebra. The PLM produced dramatic gains for virtually all students on this task. As shown in Figure 8, accuracy changed from about 57% on initial learning trials to about 85% at the end of PLM usage. Response times per problem were reduced by about 55%, from nearly 12 sec per problem to about 7 sec, suggesting the development of fluency in processing symbolic structure of equations.

Transfer to actual algebra problem solving showed equally important results. Solution accuracy did not change reliably; however, given pre-test levels of about 80% correct, we did not expect that it would. However, students maintained or slightly improved their solving accuracy after PLM training while demonstrating (as shown in Figure 9) a dramatic reduction in solution time -- from about 27 sec per problem to about 12.5 sec per problem. A delayed post-test showed that these gains were lasting: average performance was actually slightly faster when tested after a two-week interval.

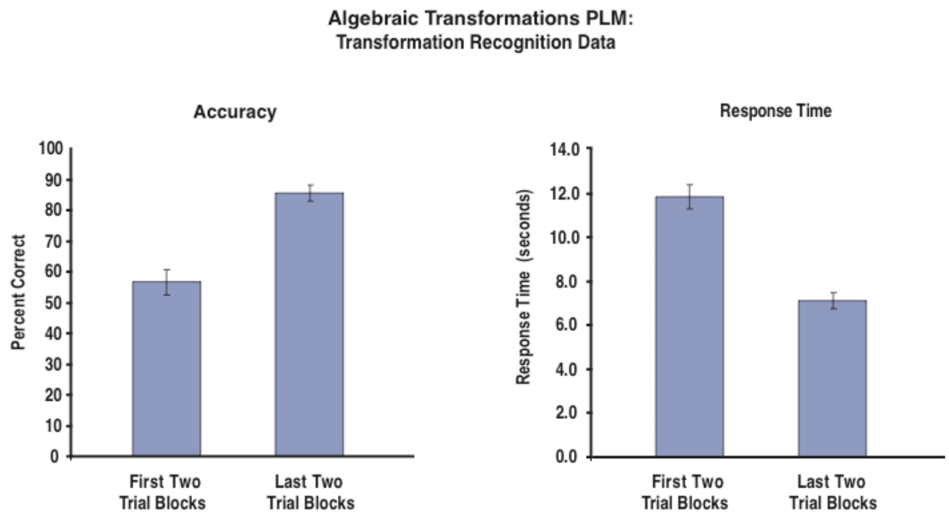


Figure 8. Transformation Recognition in the *Algebraic Transformations PLM*. Data are shown for the task students performed in the PLM: choosing the equation that was a valid transform of a target equation. First and last two trial blocks (10 trials per block) of the learning session are shown. Error bars show ± one standard error of the mean.

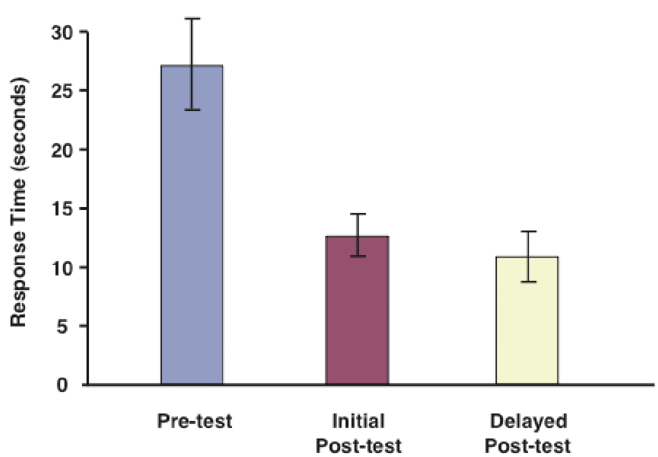


Figure 9. Fluency Data from the *Algebraic Transformations PLM*. Mean response time for solving simple algebra equations is shown for the pre-test, immediate post-test, and delayed post-test. Response times were included only for trials on which the student answered correctly. Delayed post-test occurred about 2 weeks after the end of PLM use. Error bars show ± one standard error of the mean.

The data indicate that the *Algebraic Transformations PLM* addressed learning dimensions that simply fall outside the reach of traditional instruction. According to teachers, students, and the official curriculum, these students had had considerable instruction and practice on solving equations. Yet, the combination of classwork, homework, and web activities had not done much to address the fluency and pattern recognition bottlenecks seen in early algebra learning. Against this background, the rapid gains produced by two 35-minute PLM sessions are truly remarkable. These gains appear to be enduring, not transient. Informally, we obtained effusive feedback from classroom teachers who described noticeable improvements in their algebra classes weeks after the intervention.

Future Directions. A complete algebra product will incorporate parts of the existing research prototype. We are already building this product to have multiple levels, as our research suggested that that problem of seeing in algebra is even more difficult than we suspected. Students' first use of our technology in algebra should begin with even simpler expressions, equations, and transformations than we have tested so far. Other research combining sequencing with perceptual learning methods suggests that we will see even stronger learning gains from tracking and sequencing transformation types in each learning session. A student may be competent with problems requiring subtracting a term from both sides of an equation but not with dividing a term. Or, a student may be capable where a single operation can solve an equation but not yet be comfortable or fluent where more than one operation is involved. We are currently building out the *Algebraic Transformations PLM* to include powerful features that ensure that students' learning is focused where it is most needed and that all transformation types and levels are mastered. We believe this approach is unprecedented and will lead to much greater student achievement and confidence, as well as teacher satisfaction.

SUMMARY: ALGEBRAIC TRANSFORMATIONS PLM

- **ALGEBRA I STUDENTS AT MID-YEAR UNDERSTAND THE BASICS AND CONCEPTS FOR SOLVING SIMPLE EQUATIONS, YET TAKE 27 SEC PER PROBLEM!**
- **OUR PLM INTERVENTION FOR TWO 40-MIN SESSIONS, LED TO IMPROVEMENTS IN RECOGNIZING VALID TRANSFORMATIONS AND ALSO PRODUCED SUBSTANTIAL GAINS IN PROBLEM SOLVING FLUENCY, *AVERAGING* NEARLY A 60% REDUCTION IN SOLUTION TIMES.**
- **THESE GAINS IN FLUENCY APPEAR TO BE LASTING: THEY WERE FULLY PRESERVED ON A DELAYED POST-TEST 2 WEEKS AFTER PLM USE.**

D. Solving Problems with Fractions

Understanding fractions and being able to use concepts related to fractions in problem solving are notoriously difficult areas in the elementary and middle school curriculum (Behr, Lesh & Post, 1983; Hart, 1988; Thompson & Seldhana, 2003). Many students never achieve more than minimal (and error-prone) performance levels in carrying out basic operations with fractions, and their conceptual understanding of fractions is too fragile to support transfer and application in problem solving situations. We have conducted a series of studies exploring the use of perceptual learning principles to enhance learning related to fraction problem-solving. Among the themes explored by these studies are (1) helping students learn to discriminate different underlying problem structures involving fractions; (2) enabling students to map across different representations that are related to the same mathematical structure (e.g., word problems, graphic displays, and number sentences); and (3) investigating ways to best integrate complementary modes of instruction, such as whole-class and small group instruction with individual use of PLMs.

An area of particular concern in the mathematics curriculum is students' persistent difficulty in solving open-ended word problems that involve rational numbers. These kinds of problems are staples on standardized tests and show especially large performance gaps among lower and higher-achieving students. Neither traditional instruction focused on calculation nor reform curricula focused on conceptual understanding have achieved consistent success in closing this achievement gap.

Consider the following word problems, which are among the simplest fraction problems students might encounter in this format:

1. A baseball team won $\frac{1}{5}$ of the games in the season. How many games are in the season if they won 30 games?
2. A class goes on a field trip and $\frac{1}{5}$ of the students have disposable cameras. How many students have disposable cameras if there are 30 students in the class?

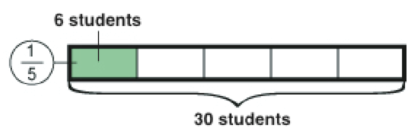
To many students, these problems look the same: the numbers are the same and the phrasing is very similar. Students often solve them in the same way, arriving at the same answer for both. Across a set of such problems, they might score, at best, about 50% correct. In fact, these problems have contrasting mathematical structures. In problem one, the fractional part is what is known and the total quantity is what the student is asked to find (answer = 150). This question might be rephrased as "30 games is $\frac{1}{5}$ of how many games?" We call this kind of problem a "find the whole" problem. In problem two, the total quantity is known, and students are asked to find the fractional part (answer = 6). This question could be rephrased as "How many students is $\frac{1}{5}$ of 30 students?" This type of problem is a "find the part" problem.

If students could reliably discriminate between these two problem structures, we would expect their performance on open-ended fraction problems to improve. Often students pick up or are taught short-cuts, such as "*of* means multiply" or "divide the whole number by the denominator of the fraction." In the problems above, these strategies are of little help. How, then, can we get students to discriminate between key problem structures?

A related challenge is helping learners track fraction concepts across different representations. For example, the two word problems above map to different graphic representations:

1. A baseball team won $\frac{1}{5}$ of the games in the season. How many games are in the season if they won 30 games?

2. A class goes on a field trip and $\frac{1}{5}$ of the students have disposable cameras. How many students have disposable cameras if there are 30 students in the class?



If students are successfully encoding the underlying problem structures, they should be able to discriminate between the two structures in either the word problem or the fraction strip format, and they should be able to match each word problem to its corresponding fraction strip. These abilities are important, not only for students' extraction of key structure but for their fruitful use of representations, such as fraction strips, which are often provided to help students but which themselves can be sources of confusion.

The general hypothesis that we have tested in our studies is that instruction based on perceptual learning principles is an effective and efficient way to achieve this kind of learning. What is more, we claim that this structure discrimination is *the central challenge* in solving open-ended word problems successfully: students who see the underlying problem structure should be able to develop a problem solution with relatively little difficulty. Thus practice that is aimed at structure discrimination should *transfer* to open-ended problem solving (provided the learning task effectively targets relevant structure). (To those familiar with the research literature on transfer of learning, especially among low-achieving students, this might be recognized as a bold and risky claim (Bransford & Schwartz, 1999).)

To test this approach, we developed and tested a series of perceptual learning modules that engaged students in classification trials designed to enhance their ability to distinguish between underlying problem structures as well as their ability to recognize and match those problem structures across different representations (word problems, graphic representations, and number sentences used to calculate solutions).

Examples of the kind of task that the students performed on each trial are illustrated in Figure 10 below.

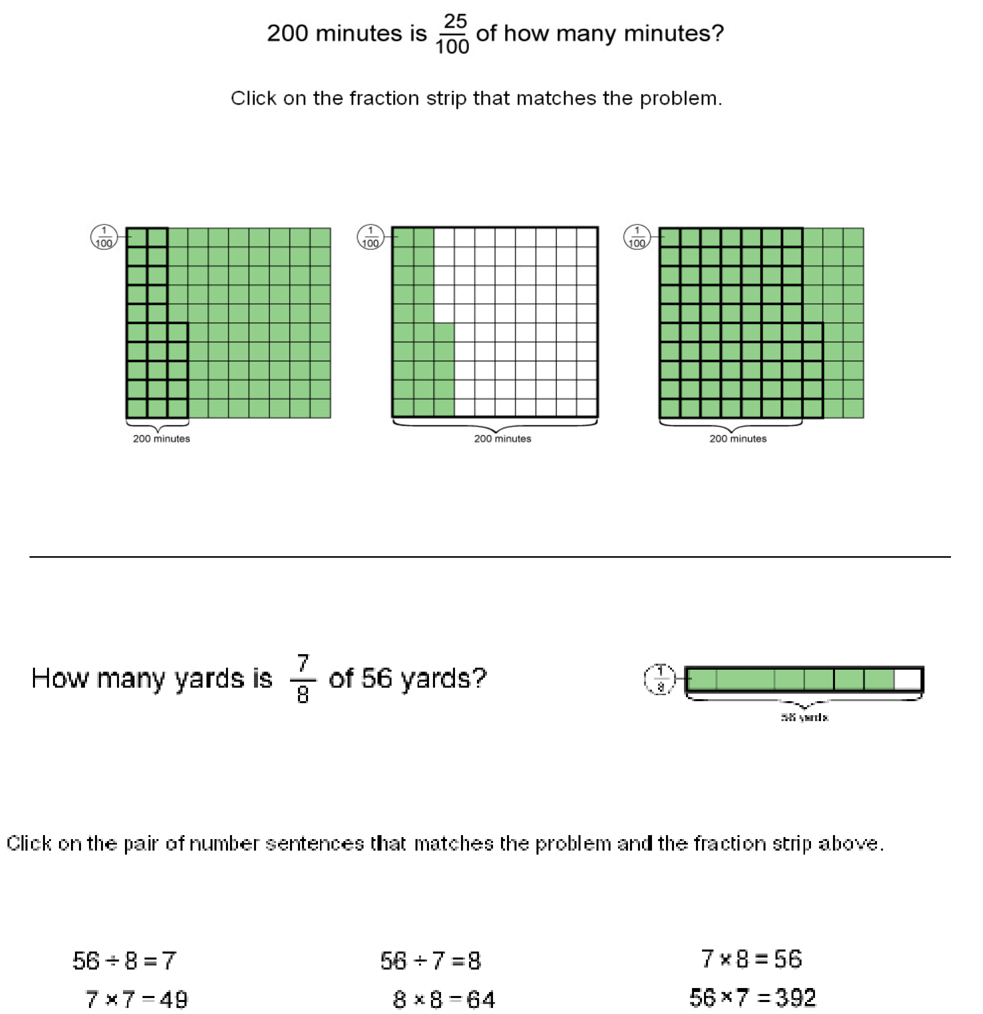


Figure 10. Sample Trials from a *Fractions PLM*. Two different types of learning trials are illustrated. Top: The learner's task is to select which of three fraction strips matches a simply stated word problem. Bottom: A simple word problem and its corresponding fraction strip are given and the learner's task is to select the pair of number sentences that would correspond to the calculation of a solution to the question.

Results. Students who practiced problem discriminations and representation mapping with PLMs performed just as well on post-tests composed of open-ended word problems as students who spent the same number of computer-based sessions directly practicing open-ended problems. However, the students indicated that using the PLM software was much more appealing than the other form of computer-based practice, and they were more willing to use the PLM software for multiple sessions. The students, who were low-income minority students in a school in which most students perform at the below-basic level on statewide standardized tests, improved from 23% correct on open-ended fraction problems to 49% correct after using the PLM software—a statistically reliable result ($p < .001$).

Combining Learning Modes: PLMs and Classroom Instruction. Our claim that PLMs address neglected dimensions of learning in no way implies that conventional instructional methods are not important. A more comprehensive model of learning would combine instructional methods based on declarative knowledge (verbalizable facts, concepts, and procedures, such as those given in class lectures, discussions, textbooks, and tutorials) with perceptual learning methods that develop attention to key features, recognition of invariant structure, and fluency.

Accordingly, an important research and development objective is to determine how best to integrate PLMs with conventional materials and classroom instruction. In our fractional reasoning project, we have begun to investigate these integration issues. In a recent study, we combined PLM interventions with a sequence of classroom instruction. A multi-week curriculum unit consisting of a series of classroom lessons, small group work, and homework practice was developed that emphasized structural relationships in fraction concepts and the notations and formats used to represent them. It also developed important conceptual relationships among fractions and multiplication and division concepts. There were three treatment groups in the study. Students in all three groups participated in the classroom instruction. Students in two of the three randomly assigned groups went on to use one of two different PLMs designed to complement the curriculum unit. The third group served as an instruction-only (no PLM) control group.

The two PLMs differed in how problems involving unit fractions (i.e., fractions with a numerator of 1, such as 1/6) and non-unit fractions (e.g., 3/8) were introduced. Students assigned to work with the "Unit First" PLM mastered unit fraction problems before being introduced to non-unit fractions. In a second phase, their PLM mixed unit and non-unit fractions together. Students in the "Mixed" PLM group were presented with unit and non-unit fraction problems from the beginning. This manipulation was designed to test competing strategies for structuring the introduction of related concepts: one approach, which is common in many educational settings, involves mastering a simpler concept before building to a more complicated one. The second approach, which has emerged more from research in memory, motor learning, and professional training settings (e.g., Schmidt & Bjork, 1992), advocates engaging learners in training that is equal in complexity to the end goals of the learning.

Results. As shown in Figure 11, results from this study favored students in the "Mixed" PLM condition, who showed the greatest learning gains and best retention from pre-test through delayed post-test. Learning gains were seen in all groups, as might be expected given the careful design and use of small group instruction, even in the "control" group. Crucially, however, these results indicate that PLM training enhances learning beyond what students achieve in a carefully designed curriculum unit relying on traditional modes of instruction. The findings also provide support for mixing problem types throughout learning, rather than teaching components separately first. Both of these issues, of course, deserve attention beyond this initial study.

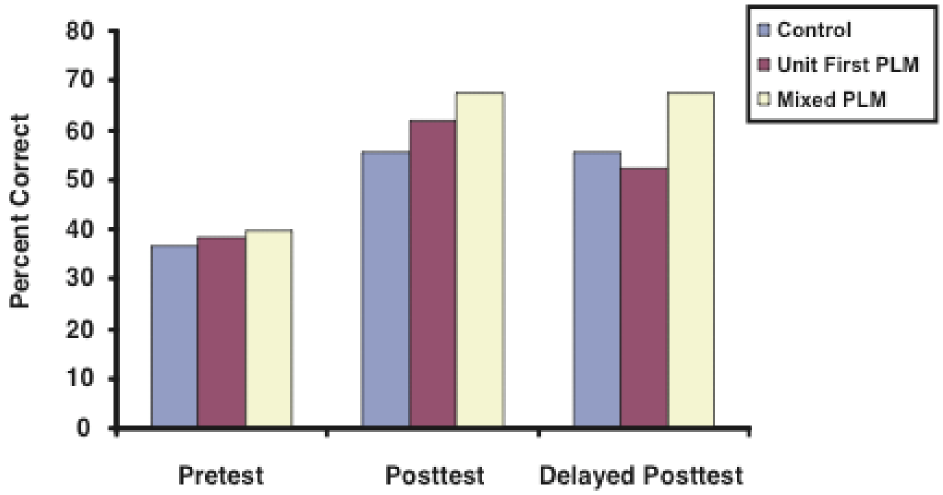


Figure 11. Results from a Study Comparing Conventional Instruction Alone to Two Conditions Integrating Conventional and PLM Components. Students completed a pre-test before participating in any instruction. The immediate post-test followed completion of the curriculum unit (including PLM training for the two PLM groups). A delayed post-test, administered 9 weeks after the end of instruction, tested retention of learning. The Control Group had conventional instruction only. The Unit First PLM Group had conventional instruction plus a PLM that established mastery for unit fractions prior to non-unit fractions. The Mixed PLM Group had a PLM that included both unit and non-unit fractions from the start.

SUMMARY: *FRACTIONS PLM*

- ➔ **PLM TRAINING AIMED AT DIFFERENTIATING PROBLEM TYPES AND MAPPING SIMILAR STRUCTURES ACROSS DIFFERENT REPRESENTATIONAL FORMATS RELIABLY LED TO SUCCESSFUL TRANSFER ON OPEN-ENDED PROBLEMS THAT ARE KNOWN TO BE DIFFICULT FOR MANY LEARNERS.**
- ➔ **PLM TRAINING WAS AT LEAST AS SUCCESSFUL AS DIRECT PRACTICE SOLVING OPEN-ENDED PROBLEMS, WHICH INDICATES THAT STRUCTURE RECOGNITION AND MAPPING IS A CORE COMPONENT OF PROBLEM SOLVING AND THAT IT CAN BE SUCCESSFULLY TAUGHT USING PLM TECHNOLOGY.**
- ➔ **STUDENTS PREFERRED PLM TRAINING TO TRADITIONAL PRACTICE SOLVING MATH PROBLEMS.**
- ➔ **PLM TRAINING COMPLEMENTS AND ENHANCES LEARNING RESULTS BEYOND THOSE OBTAINED WITH CONVENTIONAL INSTRUCTIONAL MODES ALONE.**

E. Perceptual Learning Modules in Chemistry

The focus of this document is on applications of PLMs and sequencing technology to mathematics learning. These technologies, however, have vast potential applications apart from mathematics, in primary and secondary school subjects such as science, history, geography, reading, and foreign language learning, to name a few (as well as in commercial and professional training applications). As an example of research indicating the efficacy of PLMs outside of mathematics, we briefly consider a pair of Chemistry PLMs that we have developed and studied, in collaboration with Prof. Arlene Russell (UCLA Chemistry Dept. and Natural Sciences Learning Center).

Our initial PLM in chemistry ("*3-D Angles PLM*") aimed to help university chemistry students develop structural intuitions about molecules and constraints on their formation. These intuitions are notoriously difficult to teach using conventional methods. Figure 12 shows a snapshot from the *3D Angles PLM*. On each trial, a rotating molecular model appeared, and the learner was queried about some important structural aspect (bond angle between two highlighted atoms, bond number for a highlighted atom, or hybridization). An early study showed that brief (20 - 30 minute) practice with this module reliably improved students' midterm scores in a basic chemistry course at UCLA (Russell & Kellman, 1998).

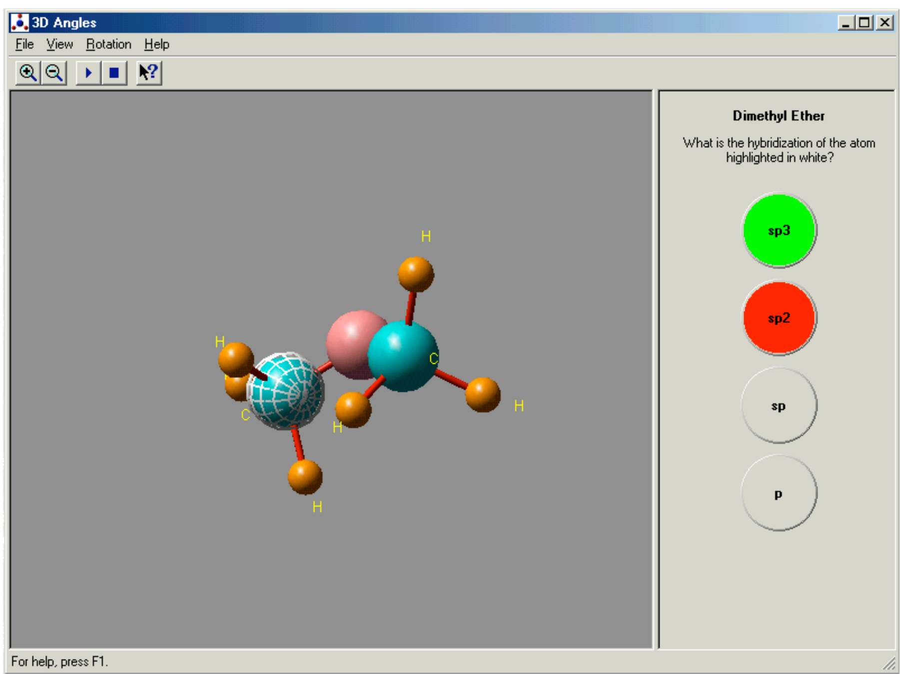


Figure 12. Snapshot from *3-D Angles PLM*. The actual PLM display showed a rotating molecular model on each trial. The learner's task was to answer a query relating to some structural aspect of the molecule shown.

A second *3-D Representations PLM* addressed mapping between 3D molecular models and conventional (but impoverished) 2-D notation systems used in chemistry to depict molecular structure. Because these notation systems often literally do not contain adequate information to specify the 3-D structure of a molecule, they must be interpreted using a combination of the information given *and* the observer's intuitions about chemical structure and constraints. Again, these skills are remarkably hard to attain from lectures or other conventional teaching methods. Anecdotally, their difficulty contributes strongly to the idea that only some students have the necessary "knack" for chemistry and that true proficiency must develop over a long period of time in the field.

Participants in the *3-D Representations PLM* were UCLA students taking a standard course in organic chemistry. In the PLM, students viewed rotating molecular models and made speeded judgments of the correctness of accompanying impoverished 2-D representations of molecular structure, as they typically are conveyed in chemistry classes and textbooks.

Although experience with the PLM lasted less than one hour, PLM students showed statistically reliable improvements on sections of the course final examination relative to a comparable group of students who did not use the PLM. One of the strongest results was that students seemed to develop intuitions about constraints on molecular structures: After PLM exposure, subjects presented with an impoverished two-dimensional representation were better at drawing (using "line-wedge-dash" representations used to depict 3-D structure) the correct 3-D structure of molecules (Russell & Kellman, 1999).

IV. Research on Optimal Sequencing Algorithms

A. Overview

Among known and imaginable innovations in computer- and web-based learning, we know of none that is likely to have broader and deeper impact than Insight's *optimal sequencing technology*. Given any learning situation in which there is more than one thing to be learned, this technology applies. It applies equally well to the learning of memory items, concepts and classifications, and procedures.

Sequencing accomplishes many learning goals, unlocking the true power of digitally-based learning. If certification of learning is important, this technology ensures that learning is comprehensive, i.e., that all of the items or concepts have been mastered. It adapts to the individual learner and focuses learning time where it is needed most. The system retires items that have been mastered according to specific learning and retention criteria. It provides continuous, objective measures of speed and accuracy, and allows for interruption and resumption of learning sessions. It excels equally in initial or efficient refresher learning. It cuts learning time in half. In combination with the perceptual learning techniques described earlier, it provides an especially powerful framework for discovery and learning of patterns in any domain.

Basic Framework. The primary question motivating sequencing technology is: Given a set of n items (or concepts) to be learned, what is the most efficient way to achieve the learning of the set for each individual learner? Some prior learning systems have been adaptive in that they vary the material presented based on the learner's accuracy. Although helpful, such approaches are incomplete. Accuracy alone gives only a partial picture of the progress of learning. To understand information processing during learning and progress toward learning goals, use of response time data is also crucial. There is a difference between a slow, hesitant (or guessed) correct answer and a confident, quick, fluent one. Tracking response speed throughout learning is a key to understanding the nature of the learner's processing, the growth of fluency in understanding, and the determinants of long term retention. Insight has recently received a Notice of Allowance for its US patent on learning systems that use the learner's speed and accuracy on prior trials to determine what comes next in a learning session.

We omit here the complexities of sequencing algorithms, their adjustable parameters for different learning situations, and so forth. The basic idea, however, is that all items (or categories) to be learned have priority scores that change dynamically after every trial, based on the learner's performance data. For every item, values related to accuracy, speed, and number of trials since last appearance, among other variables, are incorporated into its priority score. These scores are used by the sequencing algorithm to determine which item should appear next in a learning session. Also important are scoring algorithms that continuously track the learner's progress, present motivational feedback (separate from trial feedback) and retire items that are well-learned according to objective, preset learning criteria. Use of tracking and retirement of all items (or categories) ensures that learning time is focused where it is needed most, and that all items or concepts are mastered relative to objective criteria.

Implementing Laws of Learning. To make learning most efficient in terms of speed and durability, it is necessary to optimally utilize a number of laws of learning. To do this requires changing the priority of an item's selection in lawful ways as it becomes better learned, as evidenced by the accuracy and speed of learner responses. We do not elaborate these laws here but note that some are not generally known or intuitive. For example, in learning memory items, an item should not repeat on successive trials, as the answer (assuming feedback was given) will still be in short-term memory. (Long term learning strength requires exercising retrieval from long-term, not short-term, memory.) As the fluency of learning increases, the retention interval (time and/or elapsed trials before the item is tested again) should be "stretched" (lengthened). Note that these learning guidelines work against the simplest way of retiring items, which might be to present a given item several times in succession. Insight's technology incorporates an optimal (and parameter-adjustable) balance between speed of learning and durability. These and other laws of learning are used to automatically update priority scores, track achievement of learning criteria and item retirement, etc., so that the dynamically determined sequence of presentation comprises an optimal path to complete learning for each individual learner.

Research and Development. Development of Insight's optimal sequencing algorithms has taken place over a considerable period of time. A number of informal studies with memorization of math facts and foreign language vocabulary has consistently produced the result that, relative to random presentation, sequencing produces a 50% savings in learning time, with equivalent or better retention. Recently, we have begun carrying out more formal research studies; results to date have been consistent with this general observation, as quantified in an overall efficiency measure. (See below.)

B. Learning Basic Math Facts

One of the least appealing parts of learning in mathematics and other domains is that certain items must be memorized. In school mathematics, mastery of multiplication tables is often mentioned, but memorization of single-digit addition (e.g., 8 + 5 = 13), simple subtraction (e.g., 17 - 8 = 9) and basic division (e.g., 24 / 6 = 4) are also crucial. Insight is applying its sequencing technology (and also certain perceptual learning concepts) to its Best Basic Math™ product, which we believe will be the most efficient way ever devised to teach basic math facts, ensure the completeness of learning, and provide simple, effective, and individually targeted refresher learning as needed.

In developing sequencing technology, one testbed we used involved older subjects (high school and college students). We wanted to put these young adults in a situation analogous to that of a child learning multiplication facts. Children may know enough about arithmetic to reason their way to an answer, when they do not have it memorized (e.g., 4 x 4 could be done as "4 + 4 is 8, + another 4 is 12, plus 4 is 16"). But the goal of learning is to get these facts to be automatic, so as to allow their fluent use in other tasks. For adults, we formed lists of "squares" that would not commonly be in memory (e.g., 26 x 26, or 57 x 57).

Participants received trials showing a problem and were instructed to type in a solution as quickly as possible. Learning proceeded to speed and accuracy criteria. Participants were tested in three groups (14 per group): random presentation, with no retirement; random with retirement; and sequenced with retirement. They were instructed to try to memorize the answers rather than calculate them. Participants completed the learning in one to two 45-minute sessions and were given both an immediate post-test and a delayed post-test two weeks later. Given that these items were not initially known and not very memorable, we did not expect high performance in the delayed post-test, but we used it to get some evidence of durability of learning. To preclude extensive calculation, correct answers in the post-test data were subject to a strict time limit of 7 seconds.

Results. Figure 13 shows the number of learning trials needed by each group to reach the learning criteria. As can be seen, our retirement feature alone saves substantial learning time. The slightly lower number of trials to reach learning criteria in the random with retirement compared to sequenced with retirement is a consequence of more demanding requirements in the sequenced condition, designed to produce better retention. (For example, an item can appear on consecutive trials in random with retirement, producing easy retirement but probably poor learning.)

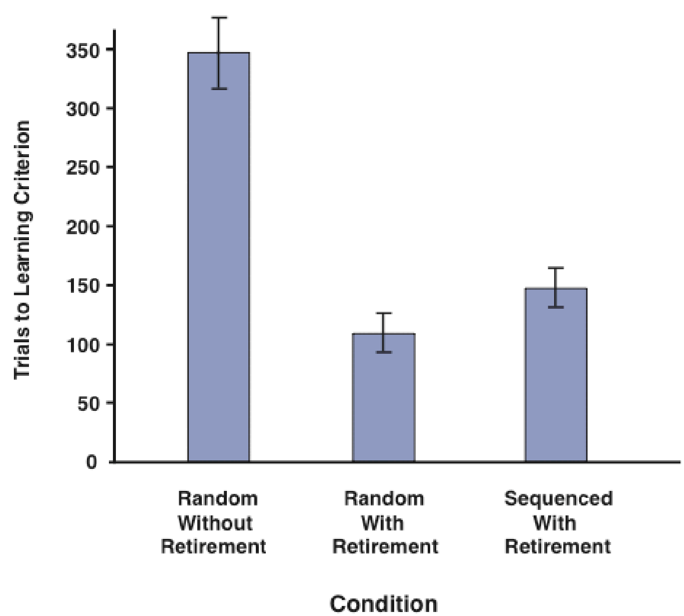


Figure 13. Trials Needed to Reach Learning Criterion by Learning Condition in Basic Math Facts *PLM*. Error bars show ± one standard error of the mean.

Figure 14 shows results from immediate and delayed post-tests. On an immediate test, the random without retirement group performed best, followed by sequenced and random with retirement. This effect on immediate test shows the effect of the vastly larger number of trials in the random, no retirement condition, in which items were not removed from the learning set even when learning criteria (for those items) had been met.

Results in the delayed post-test are generally lower, as might be expected with the nearly-meaningless items used in the study. The pattern of results is interesting, however. In the delayed test, sequenced and random without retirement show virtually identical retention, whereas random with retirement shows about half as much.

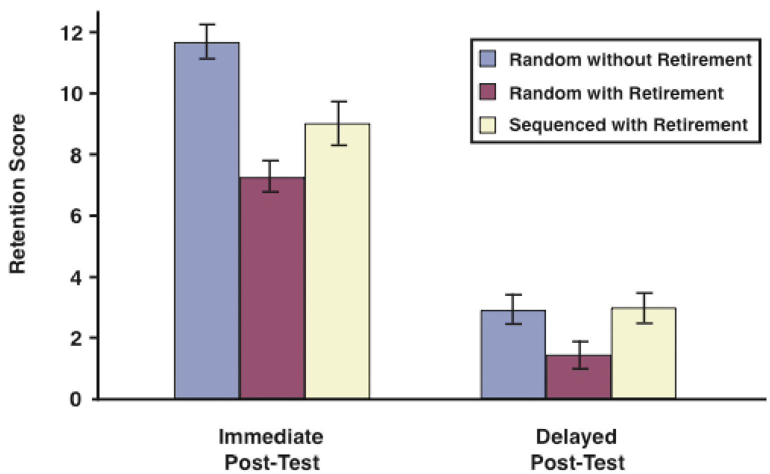


Figure 14. Retention Scores by Learning Condition in Basic Math Facts *PLM*. Delayed post-tests occurred about one week after the use of the PLM. Correct answers on post-test required both accuracy and completion within a 7-sec time limit. Error bars show ± one standard error of the mean.

The overall results in Figures 13 and 14 can be combined into a simpler efficiency measure, one that considers number of learning trials required per retained item. Assigning performance in the random without retirement condition an arbitrary value of 1, the efficiency values are 1.19 for random with retirement and 2.04 for sequenced with retirement. The results indicate important efficiencies attainable with our retirement scheme and with sequencing; in combination, they can make learning twice as efficient.

C. Studies of Sequencing for Elementary School Students Learning Multiplication tables

We have also studied sequencing for multiplication facts in with 4th and 5th graders in a low-performing urban public school. Many of these students began with poor knowledge of these facts at the start of the studies. Our results showed some of the expected advantages of sequencing.

However, for students having the greatest difficulty, there was an unsuspected but important problem. Sequencing technology presupposes that error correction (feedback) is processed sufficiently to provide at least some minimum amount of learning, on which later trials can build. For students having special difficulty or for especially difficult problems, this was did not happen reliably.

We developed and tested several feedback methods to address this problem. A comparison of auditory feedback, auditory plus visual feedback, and a specially designed interactive feedback method indicated that the interactive feedback was the most effective. These methods were tested with 77 4th and 5th graders at a public elementary school in Philadelphia, PA. Students used the learning software for three 20-minute sessions. We measured mastery, as shown by retirement of individual problems based on demanding accuracy and speed criteria: Retiring an item required perfectly accurate responses in under 5 seconds over the prior three encounters with that problem. For these subjects, initial performance was low, with fewer than 20% of problems being answered accurately and quickly. As shown in Figure 15, the interactive feedback method ("Matrix" feedback) was especially helpful in leading to greater mastery, both for relatively easy problems, which were almost perfectly mastered, and for harder problems, for which substantially greater progress was made using matrix feedback. Our interactive feedback concepts are implemented in our Best Basic Math™ product and other products under development.

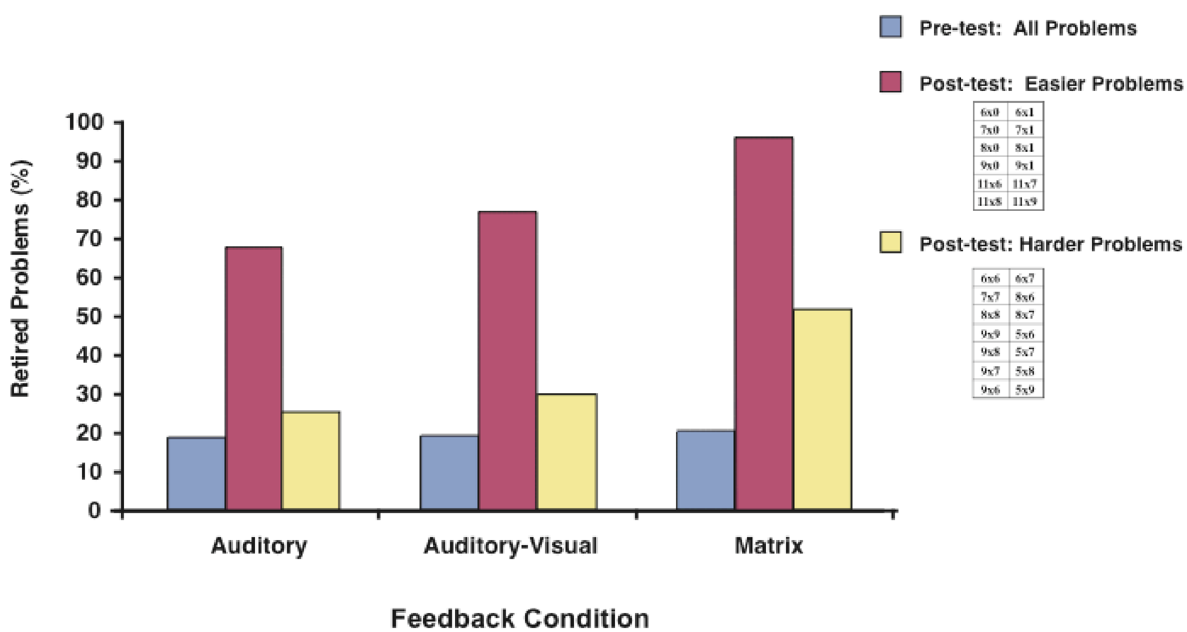


Figure 15. Basic Multiplication Fact Learning: Effects of Interactive Feedback. The graph plots the average percentage of problems that were retired by students in each of three feedback conditions. Retirement of a problem required that learning met demanding criteria, involving both accuracy and speed over multiple presentations. Matrix feedback involved proprietary, interactive feedback methods; these appeared especially helpful in overcoming special learning difficulties.

SUMMARY: OPTIMAL SEQUENCING OF MEMORY ITEMS

- ➔ **OPTIMAL SEQUENCING TECHNOLOGY FOR MEMORY ITEMS CUTS LEARNING TIME IN HALF RELATIVE TO RANDOM PRESENTATION.**
- ➔ **USE OF RETIREMENT CRITERIA IMPROVES LEARNING EFFICIENCY.**
- ➔ **SEQUENCING COMBINED WITH RETIREMENT PRODUCED TWICE THE DELAYED RETENTION OF RETIREMENT ALONE.**
- ➔ **FOR STUDENTS ENCOUNTERING SPECIAL DIFFICULTIES IN LEARNING MATH FACTS, INCORPORATION OF SPECIAL FEEDBACK STRATEGIES IN BEST BASIC MATH™ LEADS TO SUCCESSFUL LEARNING.**

D. Category Sequencing in Perceptual Learning

Insight's two primary innovation areas -- perceptual learning methods and optimal sequencing technology -- can be used separately but may offer the greatest benefits when combined. Sequencing of categories to be learned differs somewhat from sequencing of items, however. In perceptual learning, specific instances do not repeat; rather, each trial involving the learning of a certain category uses a novel instance. For example, in mastering algebraic transformations involving dividing both sides of an equation by the same term, each learning instance would be a novel one.

To date, we have done only preliminary research on optimal parameter settings for category sequencing. Although we do not elaborate fully here, there may be deep information-processing reasons why the arrangement of sequencing should be different for category learning than it is for individual memory items. Just to convey some idea of the issues here, it may be the case that in early learning, several consecutive trials involving the same structural invariants (but different particular instances) may be helpful, whereas in item learning, a given item ordinarily should not appear on consecutive learning trials.

We have developed a paradigm and carried out a preliminary study of sequencing in perceptual learning. We used learning about biological kinds—specifically beetles. It turns out there are a shocking number of types of beetles in the world—so many that it was easy to find 10 families of beetles, along with many of instances (and pictures) of each.

Figure 16. Sample Trial from *Beetles PLM* used to Test Sequencing in Perceptual Learning. On each trial, the learner made a speeded, forced choice of which picture matched the category given below. In the Sequenced condition, the algorithm used learner's response speed and accuracy to dynamically order which categories appeared on specific trials.

On each trial, a category name would appear along with two pictures, and the learner had to choose the picture that was the type of beetle named. Figure 16 shows an example. This design allowed use of blocks of 4 consecutive trials of a given concept type before going on to another. Sequencing, based on each individual learner's speed and accuracy, involved the dynamic ordering of these trial blocks.

Condition	Trials to Criterion	Number Correct in Post-test
Sequenced	232	10.0
Random with Retirement	217	8.7
Random without Retirement	437	13.4

Table 1. Preliminary Data from Study of Category Sequencing ("Beetles" PLM).

Table 1 shows the data from 39 13-18 year-olds. As in item sequencing, it appears that both sequenced and random with retirement allow completion of the learning set in roughly half the time of random presentation. Likewise, although we have not yet performed a delayed post-test, it appears that sequenced presentation may exceed random with retirement in retention. Efficiency scores for these data (assigning random without retirement as 1) are 1.4 for sequenced and 1.2 for random with retirement.

Incidentally, this study illustrates a striking characteristic of perceptual learning technology that is not shared by any other learning system we know of. Our results showed that learners became able to accurately classify wholly new instances of the 10 families of beetles after PLM use. However, as developers of this PLM, we can state unequivocally that we have little or no idea, and even less ability to articulate verbally, what *are* the distinguishing characteristics of beetles in these 10 families. Perceptual learning engages natural human learning mechanisms that extract structural invariance from experience with instances. This occurs without verbal instructions, and as long as instances and their correct categorizations are known, the creators of the learning module need not know what these invariants are. Likewise, as is known from research studies on experts in various domains, such as chess grandmasters (Chase & Simon, 1975), the successful learner may not be able to articulate fully the structures they use to be experts. This fascinating aspect of PL methods is not especially evident in mathematics learning, where we typically can specify the structures to be learned; however, it opens up a range of unexplored possibilities for a variety of other learning domains in which the structural content may be less clear. In any case, the result underscores that there are dimensions of learning, and natural, powerful human learning processes, that fall outside traditional instructional methods centered on declarative knowledge.

Although more study is needed, the data suggest that benefits of sequencing on learning efficiency will be realizable when combined with perceptual learning methods. Moreover, we believe that we have not yet determined optimal parameter settings for category sequencing and can accomplish that in further research. (For example, the optimal amount of grouping of same-category items is not yet known, nor is whether that grouping should change as learning gets beyond initial stages.) A virtue of our technology is that whatever the best parameters turn out to be, they can be easily set in our PLM Shell Program, which makes applying category and item sequencing to new modules very easy to accomplish.

SUMMARY: OPTIMAL SEQUENCING OF PL CATEGORIES

➔ **SEQUENCING TECHNOLOGY FOR CATEGORIES IN PERCEPTUAL OR CATEGORY LEARNING APPEARS TO CUT LEARNING TIME IN HALF, ALTHOUGH DATA ARE MORE PRELIMINARY AND OPTIMAL PARAMETERS ARE NOT YET KNOWN.**

➔ **COMBINING SEQUENCING TECHNOLOGY WITH PERCEPTUAL LEARNING TECHNIQUES OFFERS THE PROMISE OF ACCELERATED FLUENCY AND PATTERN RECOGNITION ATTAINED WITH MAXIMUM EFFICIENCY IN A VARIETY OF MATHEMATICS AND OTHER LEARNING DOMAINS.**

V. Concluding Perspective

As indicated by the results reviewed in this document, research and evaluation to date on perceptual learning technology and optimal sequencing algorithms has led to highly robust and consistent improvements in learning. Insight's technology addresses neglected dimensions of learning and makes learning more efficient in a variety of domains. A striking consistency is that our PLM interventions have in all cases produced strong learning effects in short time periods (e.g., two sessions of 35 minutes). This outcome suggests that major boosts in learning are possible with computer-based modules that are not too burdensome for students or teachers. The general success of these techniques suggests that they will work in a wide variety of applications in mathematics learning and elsewhere.

In mathematics learning, Insight aims to produce suites of computer-based learning modules that complement conventional instructional methods throughout primary and secondary school. Individual modules can match up with standard components of new or established curricula. It is clear that in every domain tested, such learning modules produce pattern recognition, structural intuitions, and fluency that are needed to complement classroom and textbook instruction to achieve successful learning.

Our research and development suggest that achieving learning success, especially for less talented or motivated students, requires a more detailed approach than is often realized in conventional settings. PLMs and optimal sequencing algorithms offer clear methods for successfully targeting specific learning domains and bringing every learner to an objective level of mastery. Within specific modules, tracking the various items or structures that need to be learned (such as different types of algebraic transformations) and utilizing response time as well as accuracy data in learning, ensure that learning is far more comprehensive and enduring than with other instructional methods.

Insight's PLM Shell Program allows relatively easy development of new modules targeted at specific learning domains. New modules and coordinated sets of modules are much easier to devise than is the case for most educational software. New modules inherit automatically all the features of item and category sequencing, feedback options, scoring, retirement, objective learning criteria, etc. The architecture uses flexible parameter settings that can vary to accommodate different learning domains and student needs. These features allow Insight's technology to be easily deployed, in current and future applications, to realize the learning benefits described in this report.

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