COGNITIVE PSYCHOLOGY 23, 141-221 (1991)

A Theory of Visual Interpolation in Object Perception

PHILIP J. KELLMAN AND THOMAS F. SHIPLEY

Swarthmore College

We describe a new theory explaining the perception of partly occluded objects and illusory figures, from both static and kinematic information, in a unified framework.

Three ideas guide our approach. First, perception of partly occluded objects, perception of illusory figures, and some other object perception phenomena derive from a single boundary interpolation process. These phenomena differ only in respects that are not part of the unit formation process, such as the depth placement of units formed. Second, unit formation from static and kinematic information can be treated in the same general framework. Third, spatial and spatiotemporal discontinuities in the boundaries of optically projected areas are fundamental to the unit formation process. Consistent with these ideas, we develop a detailed theory of unit formation that accounts for most cases of boundary perception in the absence of local physical specification. According to this theory, discontinuities in the first derivative of projected edges are initiating conditions for unit formation. A formal notion of relatability is defined, specifying which physically given edges leading into discontinuities can be connected to others by interpolated edges. Intuitively, relatability requires that two edges be connectable by a smooth, monotonic curve. The roots of the discontinuity and relatability notions in ecological constraints on object perception are discussed. Finally, we elaborate our approach by discussing related issues, some new phenomena, connections to other approaches, and issues for future research.

INTRODUCTION

Among the most influential aspects of Gestalt psychology was an emphasis on spatial and temporal patterns as fundamental to perception. Unlike their sensationist predecessors, the Gestaltists recognized that stimulus variables relevant to perception need not correspond to local sensations. Spatial and temporal relationships in the inputs to the senses might explain how perception can instead be in close correspondence to the outside world (von Hornbostel, 1927; Koffka, 1935). This insight sur-

This research was supported by NSF Research Grant BNS 85-19851, a James McKeen Cattell Fund Sabbatical Award and a Swarthmore College Faculty Research Grant to P.J.K. and NSF Grant 89-13707 to P.J.K. and T.F.S. Portions of this research were presented at the 28th, 29th, and 30th Annual Meetings of The Psychonomic Society (1987, 1988, 1989). We thank James Franklin and J. Edward Skeath for Appendixes A and B respectively and for very helpful discussions. We also thank Barbara Gillam, Donald Hoffman, Stephen Palmer, Dean Peabody, Dennis Proffitt, Irvin Rock, and Derek Ward for helpful comments on earlier drafts of the paper. Reprint requests should be addressed to Philip J. Kellman, Department of Psychology, Swarthmore College, Swarthmore, PA 19081.

0010-0285/91 $7.50

vives in contemporary theories, in which perceptual mechanisms are seen as detecting patterns in ambient energy that carry information about the physical world (Gibson, 1966; Johansson, 1970; Marr, 1982; Shepard, 1984).

Object perception is a paradigm case of the importance of relational information in perception. Demonstrations have been easier to develop than explanations, however. Among the most conspicuous and perplexing relational aspect of object perception is the problem of occlusion. Light is reflected to our eyes from only parts of objects, yet perception is ordinarily of whole objects. Parts of objects thus appear in our perception that have no local stimulus correlates (Michotte, Thines, & Crabbe, 1964). These facts are not often noticed in ordinary perceiving, perhaps because perception of objects despite occlusion seems automatic and accurate. In Fig. 1, for example, the unity and boundaries of the building are readily apparent. Many parts of its outer boundaries are hidden by the building itself; such self-occlusion occurs for any object viewed from any relatively stationary position. Occlusion of the building by branches of the tree is pervasive in this photograph. A count of the separate regions in the optic array bounded by projections of the tree's branches would scarcely be possible. Movement of the observer while viewing such a scene would lead to disocclusion of some parts of the layout, but also to new and complex patterns of visible surfaces and occlusion. In normal environments, there are few if any cases in which perceiving objects does not involve the problem of occlusion. Fortunately, perceivers readily detect the unity and boundaries of partly occluded objects (Kellman & Spelke, 1983; Michotte et al., 1964). In the example above, the unity and boundaries of the building were obvious, while counting the separate surface regions would be a time-consuming task requiring reflection and effort (cf. Wertheimer, 1912).

How is it that we are able to perceive the parts of objects that are occluded? In this paper we put forth an answer to this question, by elaborating a theoretical framework first sketched by Kellman and Loukides (1987). Since the theory addresses a number of unit formation phenomena besides the perception of partly occluded objects, we first describe these in Section I. Section II highlights the problems a theory of occluded object perception must surmount. In Section III, previous theories of unit formation are reviewed. In Section IV we present the basic theoretical ideas underlying our approach to unit formation, followed by a detailed exposition in Section V. Section VI takes up related theoretical issues, and recent empirical research bearing on the theory is described in parts of Sections IV–VI. Section VII explores the relations between our theory and other views of unit formation, and in Section VIII we discuss unresolved issues pertaining to the theory.

FIG. 1. Clothier Hall at Swarthmore College, partially occluded (see text.)

I. A SURVEY OF UNIT FORMATION PHENOMENA

Amodal and Modal Completion

The perception of the occluded areas of objects was termed by Michotte et al. (1964) "amodal completion," using "amodal" to refer to the absence of sensory aspects, e.g., brightness or color, in the parts of objects perceived to be behind other objects. When one views a car whose middle is occluded by a tree, the car is amodally perceived to be a single entity, yet one cannot perceive the spot of rust on the door handle. Object perception despite occlusion may be the most important case, but not the only case, in which perceived boundaries have no physical specification. Michotte et al. (1964) contrasted amodal with "modal" completion, in which perceived areas not delimited by physical differences appear with sensory characteristics. An example of modal completion is completion of perceived surfaces across the blindspot of each eye (Walls, 1954).

Illusory Figures

One type of modal completion has attracted an enormous amount of attention from researchers in recent years: the phenomenon of subjective or illusory figures. Figure 2 illustrates the most famous example, developed by Kanizsa (1955), who revived interest in this phenomenon, originally reported by Schumann (1904). The figure illustrates three perceptual effects that usually characterize this phenomenon: a central figure having clear edges even in areas where no surface quality differences exist, a depth difference between the illusory figure and (proximally) adjacent surfaces, and a lightness effect, such that the illusory figure is seen as having a different surface color from the surrounding surface. The relation between illusory figures and ordinary object perception has been unclear. It has often been suggested that illusory figures may derive from, and help to reveal, processes of object and edge perception (Brady & Grimson, 1981; Coren, 1972; Day, 1987; Gregory, 1972; Kanizsa, 1979; Rock & Anson, 1979), but few specifics have emerged. One connection, noted by several investigators, is that formation of an illusory figure is often accompanied by amodal completion of the surrounding elements. Thus, in Fig. 2, the central triangle is seen as lying atop complete circles

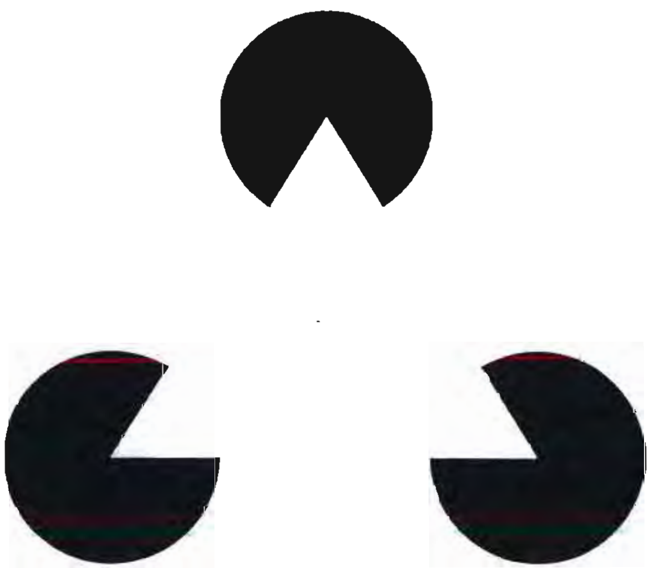


FIG. 2. A Kanizsa triangle (see text.)

(Kanizsa, 1979). Others have questioned whether accompanying amodal completion of the inducing elements is necessary for the formation of illusory figures (Day & Kasperczyk, 1983; Gillam, 1987; Kennedy, 1978a).

Apparent Transparency

A phenomenon that may be closely related to occluded object perception and illusory figures, but involves additional issues, is that of apparent transparency (Metelli, 1974; Ware, 1980). Figure 3 shows examples in the occlusion case, where all object borders are given by surface color differences, and in the illusory figure case, where they are not. In the former case, the several differently colored patches in the array are seen as two figures, one of which is translucent. In the latter case, the central area is seen to have a translucent or transparent appearance, despite the absence of any differences between it and the surround.

(a)

(b)

FIG. 3. Apparent transparency. (a) Partial occlusion case. (b) Illusory figure case.

Some Kinematic Unit Formation Phenomena

During the past two decades, there has been increasing emphasis on the role of motion in studies of object and space perception (Braunstein, 1976; Gibson, 1966, 1979; Johansson, 1975; Shepard, 1984). It now seems clear that optical transformations, given by object and observer motion, play a central role in object perception (Braunstein, 1976; Ullman, 1979; Wallach & O'Connell, 1953). Although it is known that such information can specify the unity and boundaries of partly occluded objects (Kellman & Spelke, 1983; Kellman, Spelke, & Short, 1986), there have been few studies of the principles and mechanisms involved.

Motion and Occlusion

Kinematic information for detecting occlusion was analyzed in seminal work by J. J. Gibson, George Kaplan, and their collaborators. They suggested that certain classes of optical transformations may characterize, and specify perceptually, particular classes of events in the world, such as occlusion or disintegration (Gibson, Kaplan, Reynolds, & Wheeler, 1969; Kaplan, 1969). One important source of information about occlusion is the "accretion and deletion of texture": When one surface goes behind another, projected textural details on the occluding surface will remain visible while elements on the occluded surface will gradually disappear. This information at the occluding edge is equally available to moving observers viewing stationary objects. A remarkable phenomenon demonstrated by Michotte et al. (1964) shows that an occluding edge may also be specified in the absence of surface texture. A single figure evokes perception of occlusion if its projective shape varies over time in certain ways (see Fig. 4).

These studies of the occluding edge did not directly address the perception of object boundaries in occluded regions or the question of when spatially separated visible areas are perceived as connected. Other data suggest, however, that relative motion determines the perceived unity and boundaries of partly occluded objects. Kellman and Spelke (1983) found that certain motion relationships between the visible parts of partly occluded objects specify object unity for both adult subjects and 16-week-old infants (see Fig. 5). Subsequent research has revealed a great deal about the particular aspects of motion information that govern perception in early infancy (Kellman, Gleitman, & Spelke, 1987; Kellman & Short, 1985; Kellman et al., 1986). While these studies reveal an impressive early competence to perceive partly occluded objects, it is equally clear that infant's abilities are limited in comparison with those of adults. Characterizing these limitations has been somewhat hampered, however, by the lack of a systematic understanding of the principles governing adult perception of partly occluded objects.

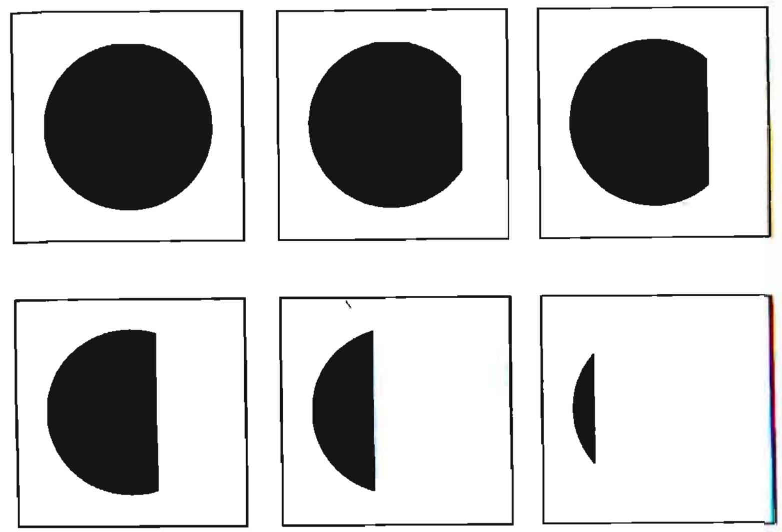


FIG. 4. Sequential views illustrating the kinetic optical occlusion phenomenon (after Michotte et al. (1964)).

Some recent observations may help to clarify the situation. It may be useful to distinguish between cases in which motion information indicates unity but not form, which we will refer to as the *primitive process*, and cases in which both unity *and* form are specified, which we will label the *rich process*. By labeling one process as "primitive," we mean simply that it specifies unity only, without indicating the locations of particular boundaries. The primitive process was illustrated in Fig. 5. Figure 6 gives examples of the rich process, in which both unity and form are readily detected.

There are four grounds for separating these two processes in object perception. First, unity can be perceived, by both adults and infants, in cases where exact form is unspecified (e.g., Kellman & Spelke, 1983). Second, perception of both unity and form in the rich process seems to require certain orientational and positional relations among the boundaries of the visible parts of objects, as is also true in stationary arrays, while perception of unity alone from motion does not. Third, there is an important difference in the conditions under which the two processes occur. The primitive process appears to require perceived motion of objects in space; it does not occur from similar optical changes given by motion of an observer viewing stationary objects (Kellman et al., 1987). In contrast, the rich process works from information given by observer or occluder motion, as in the example in Fig. 6, as well as from object

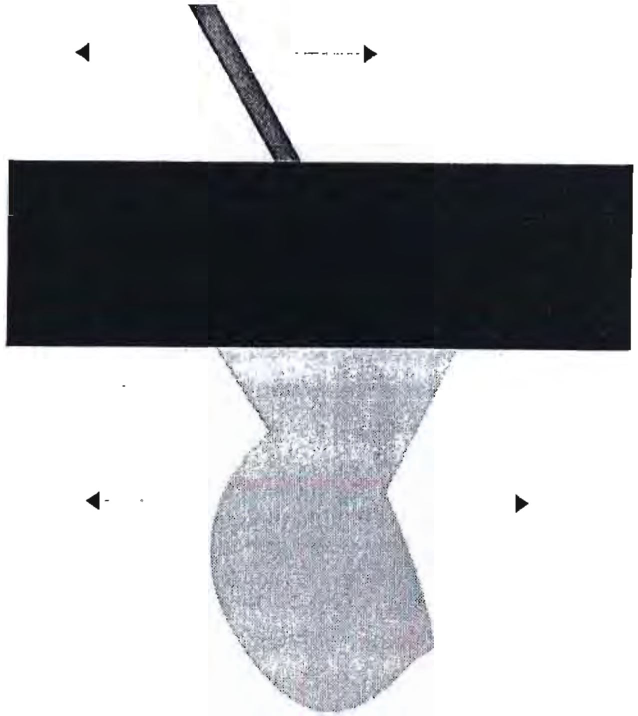


FIG. 5. Illustration of motion information for unity in occlusion cases. The two visible parts (above and below the rectangle) undergo common lateral translation. Adults and 16-week-old infants both perceive unity from this information, although the particular form of the connection between the visible parts is not given (Kellman & Spelke, 1983).

motion. Finally, the primitive process has been demonstrated early in development (Kellman et al., 1987; Kellman & Spelke, 1983; Kellman et al., 1986), while tests of relative motions without real motion of the occluded object have yielded negative results in the first half year (Kellman & Spelke, 1983, Experiment 5; Kellman et al., 1987), as have tests of static information. Tests of infant perception of kinetic illusory figures, which involve the rich process, suggest that these are also not perceived in the first half year (Kaufmann-Hayoz, Kaufmann, & Walther, 1988). The development of the rich process—including unity and form perception in both static and moving arrays—may occur during the second half of the first year (cf., Bertenthal, Campos, & Haith, 1980). Perhaps it is related to the onset of pictorial depth perception in that period (Yonas & Granrud, 1984). One likely connection involves the depth cue of interposition, which we believe has a close relation to the process of unit formation (see Section VI). Interposition and other pictorial depth cues all

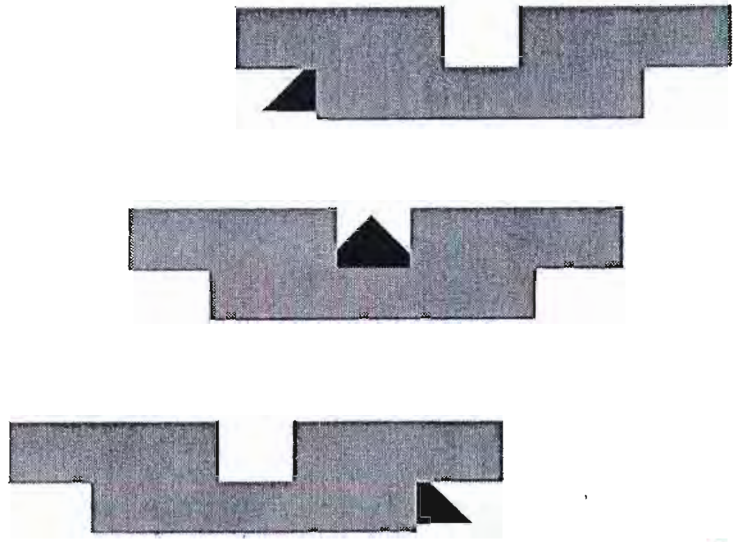


FIG. 6. Illustration of the rich process. Parts of a stationary triangle are revealed over time by the motion of an occluding object. A complete triangle with clear boundaries is perceived.

seem to become useful at approximately 30 weeks of age, which may implicate maturational factors in their development (Yonas & Granrud, 1984).

In the remainder of this paper, and in the theoretical framework developed below, we will be concerned only with the rich process. We have chosen this strategy because it now appears that a unified treatment of unity and form perception in moving and static arrays will be possible. It further appears, at present, that the primitive process is a separate way of detecting unity. Our choice of focus is not meant to minimize the importance of the primitive process, which may have the highest ecological validity in specifying unity, and may constitute an innate foundation of object perception (Kellman et al., 1987).

Kinetic Illusory Figures

Information given over time is involved not only in the perception of the occluded parts of objects but in illusory figure perception as well. Kellman and Cohen (1984) developed illusory figures based on interpolation processes across time, analogous to those operating across space in ordinary illusory figures. White figures in a black surround were shown. When sequential interruptions in the projected shapes of these figures were shown, which could all be caused by the movement of an (otherwise invisible) occluding figure, a unitary occluding figure (of the background color) was perceived (see Fig. 7).

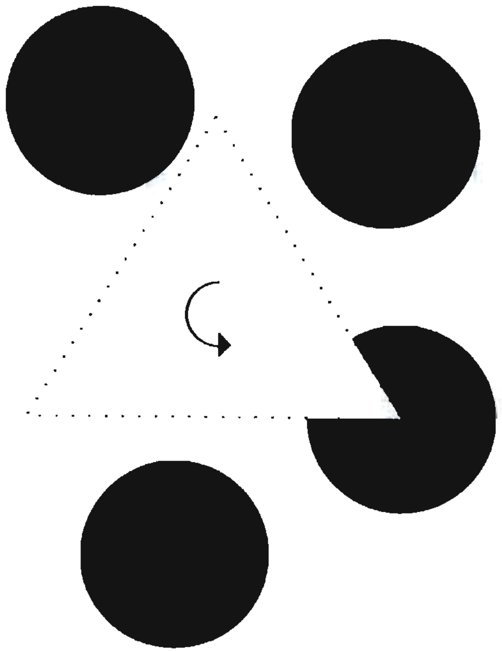


FIG. 7. Schematic of a kinetic illusory figure display. When stationary, four black circles are seen against a white background. Certain sequential changes in the circles lead to perception of a unitary central figure of the background color which moves in front of the circles (after Kellman & Cohen, 1984).

Kellman and Cohen (1984) noted several similarities between kinetic[1] and static illusory figures. Kinetic illusory figures appear at a different depth from their inducing elements. As in the static case, contour inflection points seem to require specification by physical differences, while the illusory edges run between inflection points. Finally, Kellman and Cohen noted the apparent bistability of some illusory figure displays: the central figure sometimes appeared as partly visible through holes in the surrounding surface, rather than as lying atop adjacent figures and the surround. This kind of bistability has been noted in the case of static illusory figures (Bradley, 1987; Bradley & Petry, 1977).

[1] The term "kinematic" is technically correct here, since the phenomenon involves motion patterns alone, without reference to energy. We use "kinetic," however, to refer to previously named phenomena that have come to be known by this term, e.g., "kinetic depth effect."

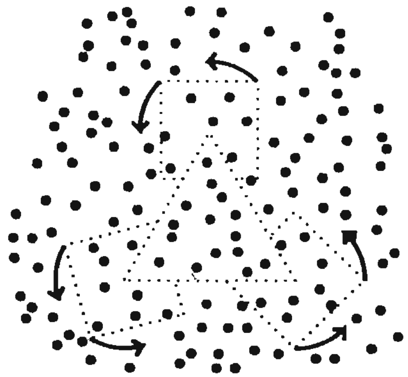


FIG. 8. Schematic of an illusory figure display with kinematically specified inducing elements (after Kellman & Loukides, 1987).

Kinematic Specification of Inducing Elements

A phenomenon of importance for several basic issues in form perception is the specification of illusory figure inducing elements outside of the luminance domain. Kellman and Loukides (1986, 1987) reported illusory figure perception from inducing elements specified over time by motion. In their example (see Fig. 8), a homogeneous field of random black dots on a white background was seen when the display was stationary. Three small squares became visible, however, when rotated (in the plane) around their centers. Although these squares were made of the same random dot texture, they became visible due to their accretion and deletion of background texture (Gibson et al., 1969). Each of these rotating elements was itself overlaid by the vertex of a central triangle. Although the central edges of this triangle were not given by any physical differences, a completely bounded, illusory triangle was perceived. Prazdny[2] (1986) independently developed similar displays. These phenomena indicate that luminance differences are incidental to the causation of illusory figures. The phenomenon of illusory figure perception from kinematically specified inducing elements converges with other evidence (Kellman & Loukides, 1987; Prazdny, 1983) in implicating a visual interpolation process sensitive to the shape and arrangement of regions of the optic array.

Summary of Unit Formation Phenomena

All of these phenomena are formally similar in that perception of

[2] We were deeply saddened to learn of the untimely death of K. Prazdny in 1987. His insights in the present context and many others have been extremely valuable. Both his intellectual gifts and personal warmth will be greatly missed.

boundaries occurs across regions where physical specification of the boundaries (by luminance, spectral, or motion information) is absent. Below we argue that the similarities, indeed identities, among these phenomena extend even further: All of these examples of visual interpolation derive from a single unit formation process, differing in ways that do not involve unit formation per se. First, we suggest some general requirements for a successful theory of unit formation, and we examine theoretical proposals previously advanced to account for one or more of these phenomena.

II. A SIMPLISTIC UNIT FORMATION THEORY

Intuitively, a first pass at the unit formation problem is to suggest that the visual system takes as units areas of homogeneous surface quality, i.e., lightness, color, and/or texture. Boundaries between units are located where changes occur in these qualities. It is useful to consider this idea, because its shortcomings are both striking and instructive.

There are a number of issues that we will not address here. For example, the present paper is not centrally concerned with the processes that detect *physically specified* boundaries. These processes are far from trivial, however. Edge detection may be based on abrupt changes in brightness, color, motion, or texture characteristics. Furthermore, some abrupt changes may occur which do not signify surface edges, such as variations due to texture, or illumination edges (Gilchrist, Delman, & Jacobsen, 1983). The scale at which changes occur is another dimension of importance: for example, Marr (1982) suggested that edge detection processes work in parallel at four levels of scale. The level(s) at which surface quality changes occur may be important in their assignment as surface edges or as variations within a single surface (Grossberg, 1987; Grossberg & Mingolla, 1985b). Research on edge detection (Leclerc & Zucker, 1987; Marr, 1982; Marr & Hildreth, 1980), lightness constancy (Gilchrist et al., 1983), edge specification by motion (Andersen & Cortese, 1990; Gibson et al., 1969) and other topics may specify in detail the inputs to the process we address here.

Our immediate concern is at the next level. Assuming some partitioning of areas based on homogeneity of surface qualities, do these areas correspond to the boundaries of objects in the world? Moreover, are these the units that observers see?

A unit formation rule based on abrupt changes in surface qualities would pick out some of the boundaries of objects. Perhaps its most interesting characteristic, however, is that in virtually every application it would be wrong about at least one object boundary.

The reason follows directly from the ecological facts that give rise to occlusion. Objects and the layout of space are three-dimensional, while the projection surface of the eye is two-dimensional. Light moves in straight lines, and most objects are opaque. As a result, when we locate the boundaries of relatively homogeneous regions in the optic array, by far the most frequent relationship between two such regions is that they project from surfaces one of which partly occludes the other. It is a rare coincidence when two objects' boundaries are tangent, and the line of sight lies in the tangent plane between them. Far more common is the (optical) interruption of a further surface by a nearer one.

Perceptually, it is the case that surface changes are normally assigned as boundaries only to one side of the change. Kurt Koffka (1935) labeled this "the one-sided function of contour." The classic figure-ground demonstrations of Rubin (1915) illustrate it (see Fig. 9.) In Fig. 9a, no information specifies the bounded side. As a result, the perceptual outcome switches over time. Figure 9b illustrates an outcome that never seems to occur. The display in Fig. 9a is never perceived as containing (simultaneously) the three bounded figures shown in Fig. 9b.

This unit formation theory, then, virtually always gives a wrong answer. The physical/ecological bases of vision imply that one surface usually continues behind a projected edge. Among the consequences of this fact is that a unitary object may reflect light to the eye from spatially separated areas, possibly a great many of them. The perceptual assignment of "one-sided" boundaries is compatible with these constraints. What is not yet clear are the processes that produce perceptual representations of the hidden boundaries and surfaces of objects. A theory of visual interpolation should indicate the sources of information for unity and boundaries despite occlusion, and should explain how such information is used by perceivers. It should also provide an account of other unit formation phenomena.

III. THEORIES OF UNIT FORMATION

A great many explanations have been proposed for particular unit formation phenomena, and some frameworks have been applied to a variety of phenomena. In this section we describe and briefly assess these theoretical proposals. Our purpose is not to be exhaustive, but to indicate the most common ways in which these phenomena have been approached.

Gestalt Approaches

Michotte et al. (1964) argued that completion occurs in accordance with Gestalt laws of organization. Their experiments suggested that past experience has little or no influence on perceptual completion. Kanizsa (1955, 1979) has also taken a Gestalt view in arguing that organizational forces tend to lead to perception of simple, regular forms. It is this tendency, applied to the inducing elements, that leads to the perception of

(a)

(b)

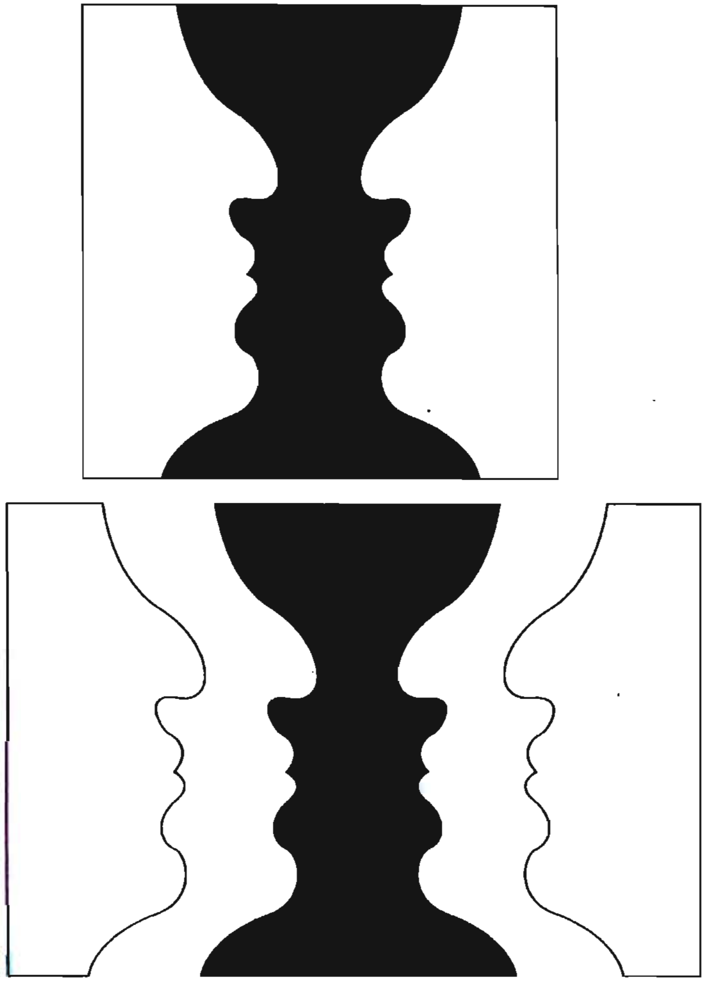


FIG. 9. Example of figure-ground organization. (a) Either two profiles or a central object are seen. The contours bound the object seen, while the adjacent surface continues behind. (b) Detached versions of the two possible bounded areas. It is difficult or impossible to see these simultaneously in (a), because the contour is ordinarily assigned as a boundary in only one direction.

illusory figures. In Kanizsa's words: "As for the singling out of the factors that determine the formation of these contours without gradients, I noted that there is one condition that is always present: the existence of parts that require completion that will transform them into more stable, more regular, and more simple figures" (1979, p. 195).

Both Michotte and Kanizsa emphasize such organizational factors in accounting for both amodal and modal completion, but the exact relation between these phenomena is left unspecified. It has also proven difficult to refine the Gestalt notions, which are said to apply to other perceptual domains as well, into principles with much predictive power. Moreover, it is possible that certain Gestalt notions, such as simplicity, confuse outcomes in perception with their causes (see below). Local processes may operate to produce outcomes of globally regular, simple configuration (cf., Marr, 1982, p. 186; Hochberg, 1978; Kanizsa, 1979; Rock, 1983). Nevertheless, Gestalt principles have had heuristic value for studies of object perception (Kanizsa, 1979; Kellman & Spelke, 1983; Michotte et al., 1964), and no successful theory of object perception can ignore the phenomena that Gestalt theory attempts to explain.

Coding Theory

Coding theory (Buffart, Leeuwenberg, & Restle, 1981) is an attempt to quantify the Gestalt notion of simplicity. It asserts that unit formation is a by-product of a perceptual system that uses a minimal code for the encoding of all scenes. For example, the number of "code entities" necessary to encode the line drawing in Fig. 10a as two differently shaped objects like those portrayed in Fig. 10b is greater than the number required to encode the scene as two overlapping rectangles (Fig. 10c). The difference in code magnitude is due to the symmetry of a rectangle. According to coding theory the perceptual system takes advantage of the redundancy of parts of a symmetric figure when it encodes the overall figure. Because irregular figures contain no such redundancy of form they cannot be encoded as efficiently as symmetric figures; hence the necessary code must be larger.[3] In Fig. 10a the "L" shape can be encoded more efficiently as a rectangle under a rectangle that as an "L"-shaped figure.

To account for perception of illusory figures, van Tuijl and Leeuwenberg (1982) proposed a variant of coding theory. They assumed that an illusory figure will be seen whenever the code magnitude necessary to encode the inducing elements as figures extending under an illusory figure is smaller than the code magnitude necessary to encode them as complete figures. Thus, a triangle is seen in Fig. 2 because encoding the inducing elements as circles under a triangle is more efficient than encoding each inducing element as a partial circle plus two straight contours.

Coding theory may improve the utility or at least the testability of a simplicity principle in perception. A daunting, unsolved problem, how-

[3] Using the coding model, a rectangle can be encoded as a line of specific orientation and length, a second line of a different orientation and length, plus a repetition of the first two lines and orientations.

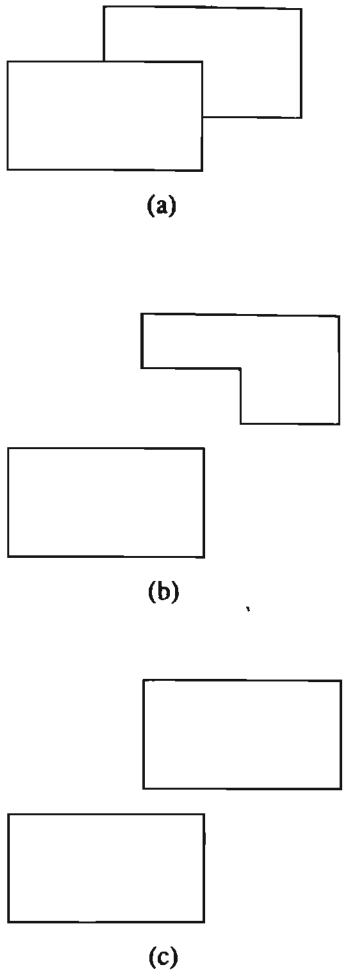


FIG. 10. Application of coding theory to occlusion. The array in (a) can be coded more efficiently as containing the two objects in (c) than in (b).

ever, concerns the process that might carry out the necessary computations needed by coding theory. It would seem that all possible configurations would have to be considered (e.g., in occlusion cases) before one could be chosen as the simplest. No solution to this problem of computational unwieldiness has been discovered, to our knowledge.

Brightness-Based Theories of Illusory Figure Perception

A number of theories have addressed illusory figures or contours, apart

from other examples of unit formation. One class of theory suggested that illusory figures are caused by processes of brightness perception (Brigner & Gallagher, 1974; Day & Jory, 1978; Frisby & Clatworthy, 1975; Jory & Day, 1979; Kennedy, 1978a). This approach has not fared well empirically and seems to have been abandoned as a general account (Day, 1987; Kanizsa, 1979; Kellman & Loukides, 1987; Parks, 1984; Prazdny, 1983; Rock, 1987).[4] It is useful to consider several of the phenomena that have weighed heavily against this general view. First, illusory figures occur in the total absence of perceived surface quality differences across the edges of the figures (Kellman & Loukides, 1987; Prazdny, 1983; de Weert, 1987). Second, illusory figure displays in which the inducing elements are defined by motion rather than by luminance differences with the background (Kellman & Loukides, 1987; Prazdny, 1986) cannot be explained by brightness-based theories. Third, kinetic illusory figures appear to have much in common with static ones; yet, they cannot be explained by appealing to processes of brightness perception either (Kellman & Cohen, 1984). Finally, attempts at a direct test of the hypothesis of brightness causation—carried out by "seeding" areas with pixels of enhanced brightness—do not produce illusory figures (Kellman & Loukides, 1987). All of these considerations indicate that illusory figures require explanation in terms of spatial (and spatiotemporal) relations of surface boundaries, rather than induced brightness effects.

Illusory Contours as the Solution to a Problem

Rock and Anson (1979; see also Rock, 1983) proposed a problem-solving view of illusory figures and contours. One of the factors responsible for perception of illusory figures, by this account, is that the inducing figures are recognized as pieces of a familiar figure. If the pieces of a display can be connected to form a complete and familiar figure, then the pieces will be perceived as parts of a single, partially occluded unit. The system may construct an occluder (the illusory figure) to "explain" why some areas are not visible. Note that such an illusory figure will only be seen when the missing parts of the familiar figure are arrayed so that they would all be occluded by it.

A view having much in common with Rock's is that of Gregory (1972,

[4] A wide variety of illusory figure and contour phenomena have been reported, most of which appear to be closely related but some of which do not. For example, induced brightness effects can occur in displays in which clear boundaries do not appear (Kennedy, 1976). We will be concerned here with all cases in which illusory figures with well-defined edges occur in the absence of complete physical specification. This includes by far the bulk of illusory contour phenomena that we know of and virtually all cases that are clearly relevant to the problem of unit formation.

1987). Gregory discusses illusory figures as an example of predictive hypotheses in perception. "Perceptually postulated" surfaces may be actively produced to account for unlikely gaps in the stimulus array.

Both Gregory's and Rock's views have intuitive appeal in suggesting some connection between illusory contour perception and the pervasive problem of occlusion, and also in emphasizing that illusory contour formation leads to simpler, more regular or familiar forms.

Illusory Contours and Apparent Depth

In an influential paper, Coren (1972) proposed a close relation between illusory contours and depth perception. Specifically, Coren argued that certain configurations function as implicit interposition cues, and illusory figures are invoked as the interposing surfaces. This view has been criticized by Rock and Anson (1979) as involving a logical paradox. Interposition presupposes an interposing surface; yet, such a surface is claimed to result from interposition. The logical problem could be avoided if certain proximal stimulus features were found to constitute information for an interposing surface; in fact, this seems to be the intent of Coren's proposal. It has proven difficult to characterize such stimulus variables, however. As noted above, Kanizsa (1979) emphasized that some figures appear to have gaps in them. Brady & Grimson (1981) attempted to refine this idea by conjecturing that tangent discontinuities on either side of a concave region may "give rise to descriptions that entail missing parts." Tangent or first-derivative discontinuities are of basic importance in our theory, elaborated below. However, we present a different view of the relation between unit formation and the depth cue of interposition. Although there is in fact an interesting connection, unit formation does not result from depth processing or detection of objects with missing parts.

Neural Dynamics: Grossberg and Mingolla's Theory

Grossberg and Mingolla (1985a,b, 1987a,b) hypothesized certain neural processes which might explain illusory figure perception. A "boundary contour" process can synthesize boundaries across physically homogeneous areas by means of cooperative interactions of similarly oriented edge detectors. A "feature contour" process is sensitive to amount and direction of contrast and triggers a filling in of surface quality between boundaries synthesized by the boundary contour process. Such boundaries are only realized perceptually if the featural quality of the area they enclose differs from adjacent areas. In its dependence on surface quality differences to explain illusory figures, this theory is a successor to earlier brightness-based theories. It incurs the same difficulties raised by demonstrations that illusory figures do not depend on such surface quality differences. These issues and a number of other aspects of the theory are examined in more detail in section VII.

IV. A THEORY OF VISUAL INTERPOLATION

Preliminary Theoretical Steps

Kellman and Loukides (1987) advanced a new theory of unit formation unifying perception under occlusion and illusory figure perception, and also relating static and kinematic unit formation phenomena. This approach has now developed considerably, and we present here a current version, some empirical tests of the model, and a discussion of issues for further research. Three theoretical postulates guide our approach, and we present them first to provide a more general perspective for the particular model advanced below. Although the current version of the model may be incomplete in some respects and perhaps incorrect in some others, we believe that these general postulates are likely to remain central as more complete models of unit formation are developed.

1. Perception of the Unity and Boundaries of Partly Occluded Objects and Perception of Illusory Figures Are the Results of an Identical Unit Formation Process

As a corollary to this claim, we suggest that the phenomenal differences between these two cases (i.e., between "modal" and "amodal" completion) are the results of factors outside of the unit formation process, specifically the depth placement of units formed. We present two lines of argument for this claim, the former consisting of "visual arguments" and the latter consisting of data.

Several phenomena suggest a very close relation between so-called modal and amodal completion. First of all, it has occasionally been noticed that some ordinary illusory figure displays can take on an alternative appearance. In Fig. 11a the white triangle is seen lying atop the other surfaces in the array, modally completed in that it has obvious surface qualities, including the well-known brightness enhancement relative to the surround. Less frequently, the black areas appear as three holes in a white surface, and a triangle appears amodally behind the white surface with its three corners visible through the holes. In Figure 11b this mode of appearance is illustrated by the addition of lines completing the boundaries of the holes or windows. The bistability of illusory figure displays has previously been pointed out by Bradley and Petry (1977), who produced the intriguing example shown in Fig. 11c. The Necker cube which normally appears in front of the other surfaces in the array can appear quite vividly floating in a dark space seen through windows. The phenomenon also occurs in kinematic cases: some subjects in Kellman and

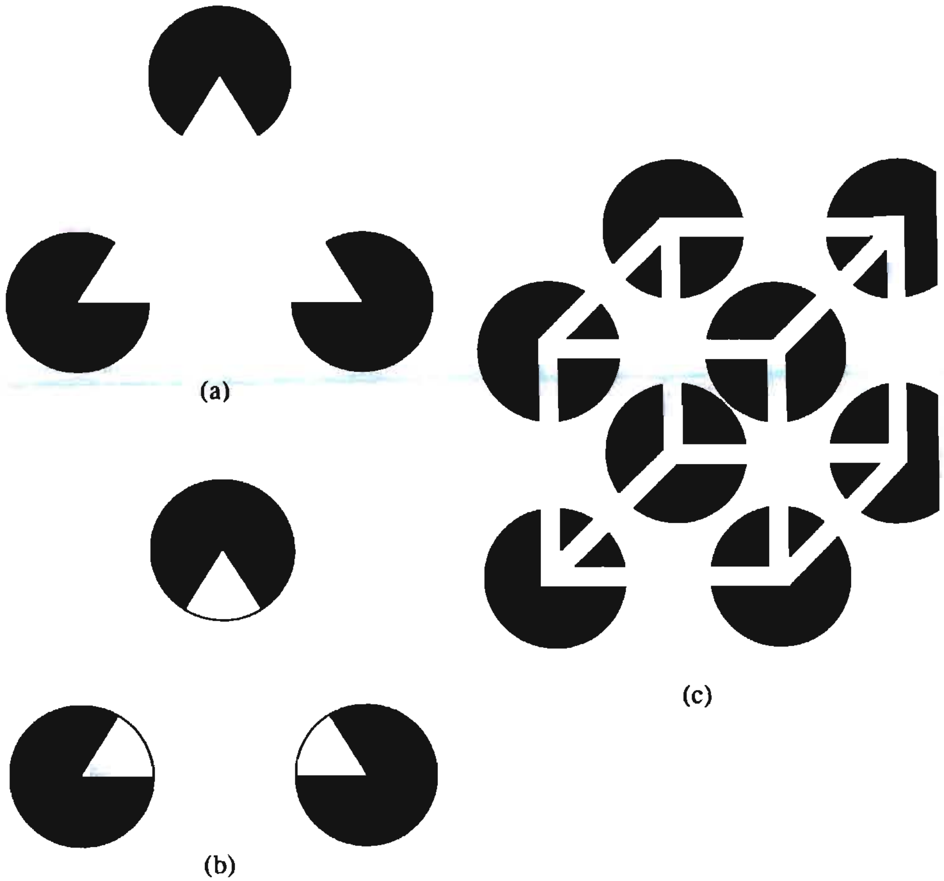


FIG. 11. Bistability of illusory figure displays. (a) Standard illusory figure display. (b) Same as (a) with lines added to emphasize possible appearance of (a) as an occluded triangle. (c) Illusory Necker cube (Bradley & Petry, 1977).

Cohen's (1984) experiment spontaneously reported amodal completion of the central figure behind the surround rather than an illusory figure in front.

A second phenomenon makes a similar point, perhaps even more directly. Figure 12 gives an example of what we call a "spontaneously splitting figure" (SSF). The phenomenon has been mentioned as an example of perceptual organization (Kanizsa, 1979; Koffka, 1935; Parks, 1986), but has received little systematic attention. There are two interesting aspects of this type of display. The first is that, although the entire figure is homogeneous in surface quality, it tends to be seen as two dis-

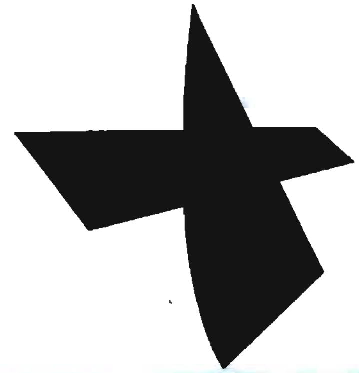


FIG. 12. A spontaneously splitting figure (see text).

tinct units. One of these figures, seen on top of the other, is modally completed; its illusory edges can be seen. The other figure is amodally completed; although its unity is obvious, its middle section is hidden behind the nearer unit. The second interesting aspect becomes evident after 30 s or so of sustained viewing. The relative positions of the two figures reverse! The figure that was modally completed now appears amodally and vice versa. This switching of position goes on indefinitely with prolonged viewing.

All of these phenomena involve switching between so-called modal and amodal completion. How should we conceptualize the relation between these processes? The answer we suggest, first proposed by Kellman and Loukides (1987), is that there are not two processes of unit formation here. The units in these various cases do not change; only their depth relations change. In Fig. 12, the units remain the same shape throughout; only their depth ordering changes with time. The differing appearance of "modal" and "amodal" completion has nothing to do with processes of unit formation, but depends on the depth placement of units formed. In the SSF case, no depth information indicates which unit is nearer, so the depth ordering vacillates.[5] In an illusory figure as in Fig. 11a, the process of unit formation produces the same triangle whether that triangle is seen atop three circles or through three holes in the surface. The process of

[5] It is important to realize that some aspects of the appearance of the various unit formation phenomena may differ, despite their origin in a single interpolation process. For example, the contours of a partly occluded object may be less vivid to inspection in some sense because they are out of sight behind another object. The boundaries of spontaneously splitting figures may appear momentarily elusive during switches of depth order. The claim of an identical unit formation process in these cases is not inconsistent with some differences of appearance due to other aspects of scenes, especially depth placement.

locating units in depth is separable from the process of unit formation. The predominant appearance of illusory figures as lying atop other surfaces is due to a weak depth cue noted by Rubin (1915): Enclosed areas tend to be seen as figures (rather than holes) whereas their surrounds tend to appear as grounds.

The claim of identity between perception of partly occluded objects and illusory figure perception has many interesting consequences, and also provides useful converging operations for the study of unit formation. One consequence is that any display in which the unity and boundaries of a partly occluded object are clearly perceived should be transformable by simple rules into a successful illusory figure display, or an SSF display. The appropriate transformations preserve the relevant edges in the scene, but differ in the way the optically unspecified area is situated. In the occlusion case, the unspecified areas of the figure lie between two visible parts on either side of an intervening area of different surface quality (the occluding object). If this display is transformed into an illusory figure display, the specified edges become part of illusory contour inducing elements, and the unspecified area lies in a homogeneous field between them. In the case of spontaneously splitting figures, the optically unspecified region and the specified edges are part of the same bounded, homogeneous region. Finally, in transparency displays, the central area differs in surface quality from all of the other regions of the display. Figure 13 gives an example.

Our informal tests of the prediction of an underlying identity among these unit formation phenomena have been highly confirming. Clear cases of unity and boundary perception in one display type predict the same outcome in the others; similarly, clear cases of unrelatedness of visible parts or of inducing elements are mutually predictable. Finally, borderline or ambiguous cases seem to be the same in different domains.

More formal tests of these claims are obviously needed, however, and we recently completed such an investigation (Shipley and Kellman, in press). One experiment consisted of two parts: a test of perceived unity in occluded figure displays and a test of perceived contour strength in illusory figure displays. Magnitude estimation was used in both tasks. In the unity task, subjects rated on a scale of 0–10 how strongly the separate visible parts appeared to be connected in displays like those shown in Figs. 14a and 14b. A display made of real objects (the four corners of a square of paper protruding from behind a book) was used as a modulus and was assigned a value of 10. Subjects were told to assign a value of 0 if there was no impression of connectedness whatsoever.

In the illusory figure task, subjects were shown a Kanizsa triangle display, and their attention was directed toward the edges of the perceived central figure. Subjects were told that edges of this clarity or strength were to be assigned a value of 10. The experimental task was to rate the strength of edges in the displays presented on a scale of 0–10, where 0 indicated that no edge was present. Figure 15 shows two of the displays used.

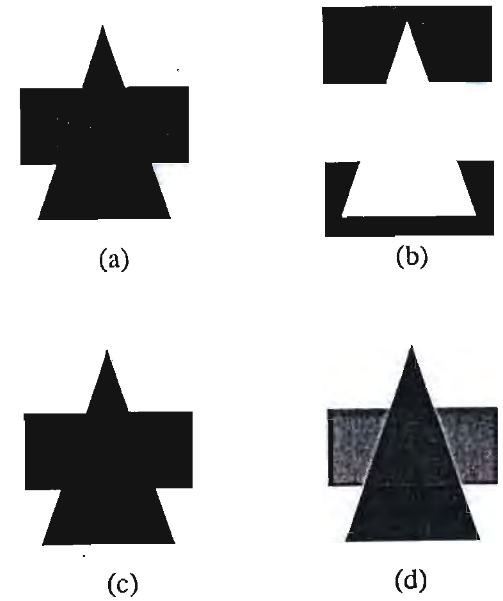
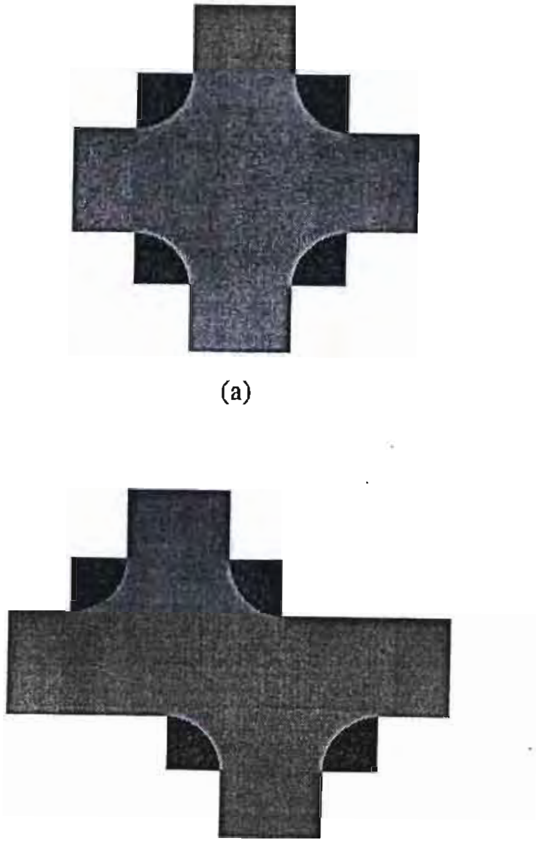


FIG. 13. Equivalent unit formation cases with differing appearances. (a) Partially occluded figure. (b) Illusory figure. (c) Spontaneously splitting figure. (d) Transparent figure.

The major independent variable in both tasks was the alignment (or misalignment) of the luminance-specified contours. Misalignment varied from 0 to about 20′ of visual angle, a value obtained through pilot testing. Figures 14 and 15 show corresponding occlusion and illusory figure displays; in each, the (a) figure has aligned edges and the (b) figure has misalignment corresponding to 20′ at the subjects' viewing distance in the experiment. Misalignments involved the vertical edges in half of the displays and the horizontal edges in the other half.

Half of the subjects did the unity task first, and the other half did the contour clarity task first. Within each task, the order of stimulus presentation was randomized. The tasks were structured so as to be superficially quite different. In one task, subjects rated the connectedness of separated parts with an intervening object in between, while in the other task, subjects' attention was directed to the robustness of illusory edges. Despite the apparent differences in the tasks, we hypothesized that if unity under occlusion and illusory contours derive from the same unit formation process, misalignment should affect both in the same way.



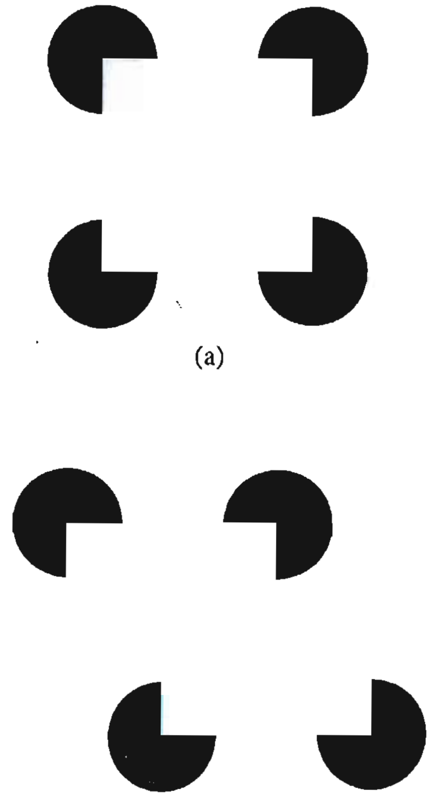
(a)

(b)

FIG. 14. Occlusion displays used to assess the effect of misalignment on perceived unit. (a) Zero misalignment. (b) Forty minutes of arc misalignment when viewed from 1.7 m.

Figure 16 plots both unity ratings and rated edge clarity as a function of misalignment. Horizontal and vertical misalignment ratings have been pooled, since these did not differ. As inspection of the figure suggests, there was a nearly perfect correlation between the mean ratings of unity and of illusory contour clarity for displays with equivalent misalignments ($r = .99$, $p < .001$). The results are highly consistent with the hypothesis that perception of partially occluded figures and illusory figures derive from the same process.

It seemed possible that this high correlation could have resulted from subjects' realizing during the course of the experiment that alignment of edges was the characteristic being varied in both kinds of displays. Al-



(a)

(b)

FIG. 15. Displays used to assess the effect of misalignment on illusory contour clarity. (a) Zero misalignment. (b) Forty minutes of arc misalignment when viewed from 1.7 m.

though the instructions and tasks used here were typical for studies in each domain (e.g., Dumais & Bradley, 1976; Kellman & Spelke, 1983), subjects might have subverted the tasks by ignoring apparent connectedness or edge clarity and simply rating the misalignment of displays.

To check this possibility, a follow-up analysis was carried out. During the first several trials of the experiment for each subject, neither the commonalities between tasks nor the uniqueness of the misalignment variable should have been obvious to subjects. Accordingly, we examined the first five trials in the experiment for each subject. For half of the subjects, these data were trials of the unity task, and for the other half, they were contour rating trials. The correlation between unity ratings and ratings of contour clarity on the first five trials (a between-subjects correlation) was .96, $p < .001$. From these data, it does not seem likely that

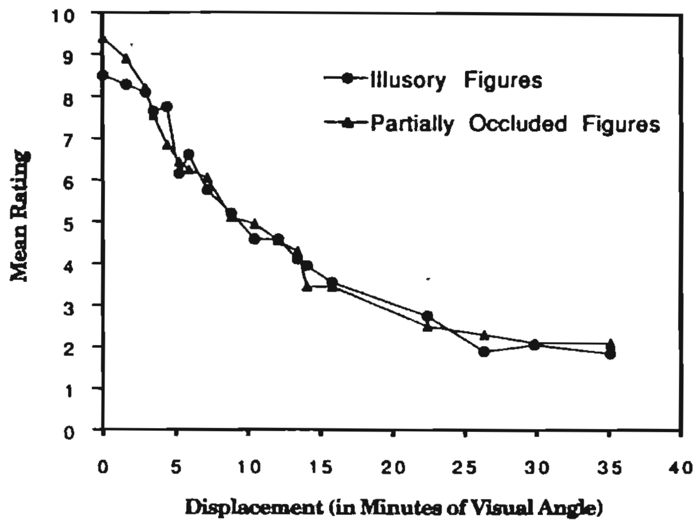


FIG. 16. Mean ratings of unity [partially occluded figure displays] and illusory contour clarity [illusory figure displays] as a function of misalignment (n = 20).

the close correspondence on these two tasks derives from any explicit recognition of the misalignment variable. Subjects' perception of unity in occlusion displays simply predicts contour clarity in illusory figure displays and vice versa.

These results join with the arguments given above in suggesting that the process of perceiving object unity despite occlusion and the process giving rise to illusory figures are one and the same. More recent experimental findings also support this conclusion (Shipley and Rellman, in press).

2. Static and Kinematic Perception of Hidden Object Boundaries May Depend on a Single Process

The many similarities between visual interpolation processes across space in static arrays and across space and time in the kinematic case suggest that these derive from the same or closely related processes. Phenomenally, both result in clear object boundaries and perceived depth differences between the perceived object and adjacent surfaces. Moreover, the identity of the unit formation process in occlusion cases and in "modal" (illusory figure) cases appears to characterize both static and kinematic phenomena. Figure 17 illustrates this parallel. The display in Fig. 6 has been altered so that what were the visible edges of a partly occluded object are now specified by interruptions in kinetic illusory

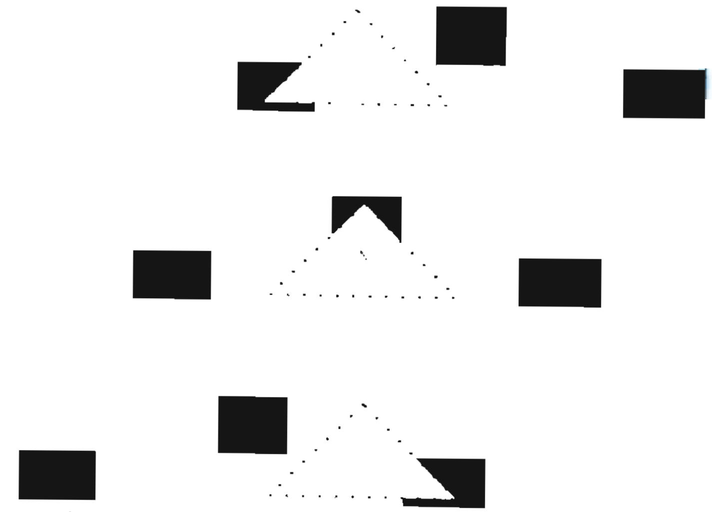


FIG. 17. Sequential views illustrating a kinetic illusory figure display equivalent to the kinetic occlusion example in Fig. 6.

figure inducing elements. The displays are interchangeable, in that the conditions that give rise to unity and form perception in the occlusion case also give rise to kinetic illusory figures.

In terms of causation, both static illusory figures (in which all vertices are specified simultaneously) and kinematic ones (in which vertices are specified sequentially) can be generated with luminance-specified inducing elements or with inducing elements specified by motion (Kellman & Loukides, 1987). The spatial and temporal cases necessarily involve some different aspects, e.g., the rules relating edges over time in the temporal case, but the many parallels suggest that these phenomena might be explained in a unified spatiotemporal framework.

3. The Mechanisms of Unit Formation Incorporate Basic Ecological Constraints, Specifically Utilizing the Information Provided by Spatial and Spatiotemporal Discontinuities in Projected Edges

What information is available to perceivers about which parts of objects are occluded and where object boundaries lie in occluded regions? Consideration of optical and ecological facts about objects and their projections is likely to be a key to understanding how the visual system detects occlusion and object boundaries. From our consideration of these aspects of the problem, we propose that spatial and spatiotemporal discontinuities

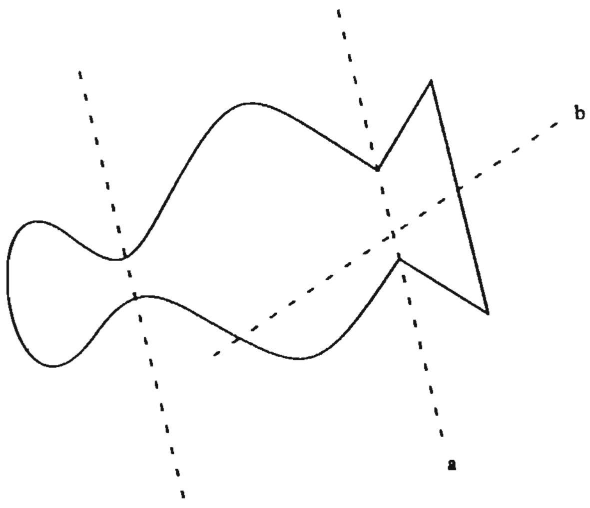


FIG. 18. Illustration of Hoffman and Richards's (1984) theory. Lines labeled 'a' indicate boundaries of natural parts of the figure. Line b indicates an unnatural part boundary.

in the projections of surface edges play a fundamental role in unit formation (cf., Brady & Grimson, 1981).

Spatially, by "discontinuity" we mean a discontinuity in the first derivative of the function describing an edge in the optical projection. A useful clue to the importance of this kind of discontinuity can be found in the work of Hoffman and Richards (1984). These investigators were interested in a problem different from, but related to, the problem of unit formation, namely what perceivers take to be the natural parts of unitary objects. For example, in Fig. 18, the dotted lines marked 'a' show a natural decomposition of the object, while the dotted line at 'b' does not. Hoffman and Richards suggest that judgments of the natural parts of objects reflect a topological fact about the interpenetration of separate objects. A theorem of "transversality" in differential topology indicates that whenever two solids interpenetrate, a first-order discontinuity will be formed where the surfaces intersect.[6]

Hoffman and Richards argued that these points of discontinuity, specifically concave discontinuities in the outer boundaries of objects, are thus ecologically sensible indicators of natural break points of objects into parts. However, for greater generality in applying the theory to objects without abrupt discontinuities, they expanded the concave discontinuity notion to include maximum points of outer boundary concavity (which may be part of smooth curves), as in the example given in Fig. 18.

The transversality notion is important for our purposes, but we make somewhat different use of it than Hoffman and Richards. Specifically, we suggest that first-order discontinuities are formed in the optical projections of objects whenever one object partly *occludes* another. This conclusion requires an additional proof, which is given in Appendix A. The requirements are quite intuitive, however. First, from projective geometry we know that whenever a discontinuity at the junction of two interpenetrating 3-D objects is projected onto a 2-D surface, it will yield a discontinuity.[7] In other words, first-order discontinuities are projectively invariant. Second, any case of partial occlusion is projectively equivalent to some case of object interpenetration and vice versa. Although this is true for both polar and parallel projection (Appendix A), it is easiest to visualize by considering parallel projection. Imagine viewing from a certain position two objects at different depths, one of which partly occludes the other. Translating one of the objects toward the other along the line of sight is sufficient to change the array from a case of occlusion to a case of interpenetration. Yet this transformation will not change the projection in any way (see Fig. 19). In short, if interpenetrating objects always lead to first-order discontinuities, then so do all cases of partial occlusion.

The visual system may make use of the fact that discontinuities in the first derivative occur in all cases of occlusion. Detecting spatial discontinuities may be the first step in perceiving partly occluded objects as unitary, at least for stationary observers viewing stationary arrays. Since such discontinuities may also occur in object boundaries without occlusion (wherever object boundaries are not smooth), detecting discontinuities is not a sufficient basis for perceiving occlusion. Below we explain the additional conditions that determine unit formation under occlusion.

When motion is involved, the notion of a discontinuity may include certain classes of change in an object's projection over time. When the projected area undergoes a nonperspective change, i.e., a change that could not arise merely from translations and rotations of a rigid object in three-dimensional space, it may comprise a spatiotemporal discontinuity. The relevant ecological fact here is that the optical changes given when one object occludes another are characteristically different from those in

[6] Strictly speaking, a discontinuity will not be formed in the degenerate case where a surface of one of two interpenetrating objects lies exactly in the same position as a surface of the other object. Mathematically, this case is handled by specifying that slight perturbations (small changes in the positions of the objects) guarantee the existence of discontinuities (Guillemin & Pollack, 1974). Perceptually, the degenerate possibility will not be important for our purposes.

[7] Again excluding degenerate cases (see Gans, 1969).

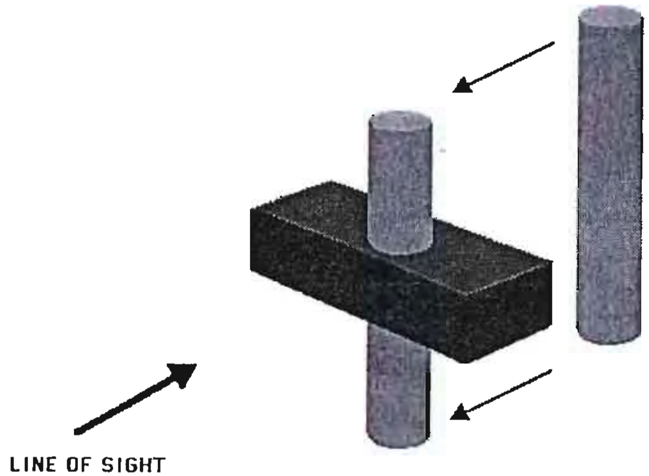


FIG. 19. Projective equivalence of occlusion and interpenetration. Under parallel projection, no projective change occurs when a case of occlusion is transformed into a case of interpenetration by translation along the line of sight.

the class of perspective changes (Gibson et al., 1969). Additional conditions involved in spatiotemporal unit formation are likewise described below.

V. PARTICULARS OF THE THEORY

The theory attempts to explain how partly occluded objects are perceived as units with definite shape. It also applies, however, to illusory figures, apparent transparency displays, and spontaneously splitting figures. A general description is that the theory encompasses boundary and unit formation in the absence of local specification. These include cases in which boundaries are perceived through optically homogeneous regions and cases where spatially separated visible areas are perceived to be connected.

I. Discontinuities in Space or Time Are Necessary Conditions for Visual Interpolation

Spatial Discontinuities

As explained above, a spatial discontinuity is an abrupt change in boundary direction of an object's optical projection (discontinuity in the first derivative). More formally, if the bounding edge of a projected region is described by $R(t)$, a vector function of a parameter t, i.e., $R(t) = x(t)i + y(t)j$, where i and j are unit basis vectors along the x and y axes, and $x(t)$ and $y(t)$ are real-valued functions of t, then a first-order discontinuity is a

point at which dR/dt is not continuous. A fundamental claim of this theory is that visual interpolation will not occur in the absence of first derivative discontinuities.

Perceptually, some extreme curvatures that are not first-order discontinuous in mathematical terms probably also function as discontinuities, or in some cases activate the unit formation process weakly. The point is really a logical one, since some first-order continuous functions with very small radii of curvature can approximate functions with discontinuous first derivatives to an arbitrary degree of precision. For example, consider functions of the form

$$R(t) = ti + (t^2 + s^2)^{.5}j, \qquad (1)$$

in which s is positive. With arbitrarily small s, this function approximates the function

$$R(t) = ti + |t|j. \qquad (2)$$

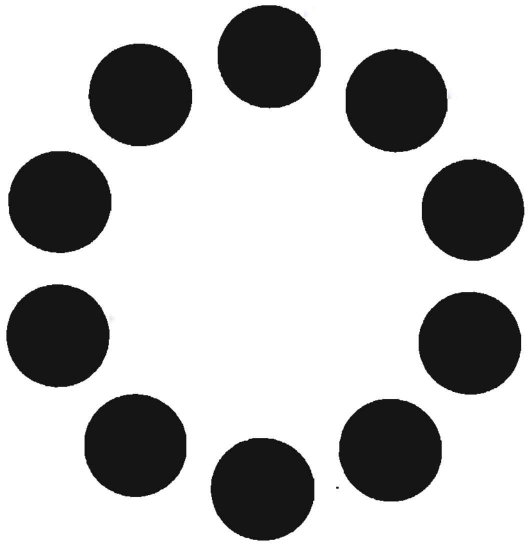
The latter function has a discontinuity in the first derivative (and all higher derivatives) at $t = 0$. The function in Eq. (1), however, has no discontinuity in the first derivative. Nevertheless, there is some value of s small enough so that the function in Eq. (1) is indiscriminable from that in Eq. (2). Note, however, that first-order continuous approximations to functions with first-order discontinuities are a highly constrained class of functions. All such approximations will have extreme curvature near the inflection points. For example, the function given in Eq. (1) has radius of curvature s at $x = 0$. Thus, if the relevant stimulus information for the unit formation process lies in first-order discontinuities, this information might be picked up by perceptual mechanisms sensitive to both actual discontinuities and points of extreme curvature.

The converse also holds. One can construct boundaries containing first-order discontinuities that are arbitrarily close in appearance to boundaries described by some first-order continuous function whose radius of curvature is relatively large. Thus, discontinuities can be made undetectable.

These logical considerations indicate that empirical specification will be needed to define the range of extreme curvatures that function as discontinuities. Registration of discontinuities seems unlikely to be an all-or-none affair; rather, it may vary continuously with sharpness of curvature. Strength of interpolation might vary with the strength of registered discontinuities.

We have been able to confirm the necessity of discontinuities for the perception of illusory figures. Subjects were much more likely to report seeing an illusory figure as well as report clearer illusory figures in displays that contain discontinuities at the ends of the interpolated contours (for example, fig. 20b) than in displays that do not contain discontinuities

(a)



(b)

FIG. 20. Display pair from the discontinuity experiments. In (a) no subject reported a clear illusory figure. In (b) subjects perceived a clear illusory circle. Both displays have identical areas tangent to a central circle; display (b) was obtained by cutting away parts of (a).

(Fig. 20a). For every display pair, at least twice as many subjects reported illusory figures for the display containing discontinuities, and no display without discontinuities was seen as having an illusory figure by a majority of subjects. These results hold for both regular and familiar illusory figures as well as irregular illusory figures (Shipley, 1988; Shipley & Kellman, 1990). These data also give some indication that the registration of discontinuities may be a matter of degree (Shipley & Kellman, 1990).

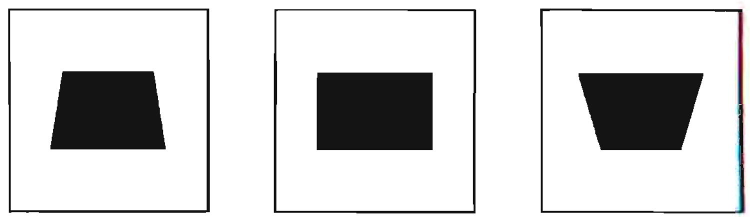
A spatial discontinuity is a necessary but not sufficient condition for unit formation. If certain other conditions are not met (see below), unit formation does not proceed.

Spatiotemporal Discontinuities

In unit formation from information given over time, the initiating conditions are likely to be projective consequences of occlusion *events*. These might include all continuous changes in the optical projection of an object that could not be caused merely by movements of that (rigid) object in 3-D space. Figure 21a depicts a projective change that could result from unoccluded movement in space, and Fig. 21b shows a change that could not.

The rationale for our definition of spatiotemporal discontinuity parallels that given previously for spatial discontinuity. Relative motion of objects,

(a)



(b)

FIG. 21. Sequential views illustrating (a) a perspective transformation and (b) a nonperspective transformation.

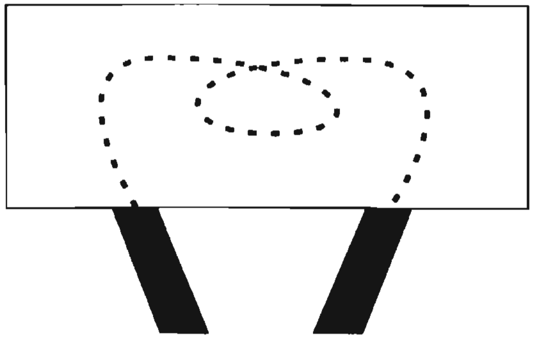


FIG. 22. Illustration of the need for a monotonicity constraint.

one of which partly occludes the other, will normally give rise to spatiotemporal discontinuities; moreover, such optical changes are highly specific to cases of occlusion (Gibson et al., 1969).

A more minimalist possibility is that occlusion events are detected because they introduce spatial discontinuities over time. The relative importance of specifically spatiotemporal discontinuities, as opposed to temporally introduced spatial discontinuities, needs to be addressed in future research.

2. New Contours Are Perceived When the Edges Leading into Discontinuities Are Relatable to Others

Spatial Relatability

Relatability, both spatial and spatiotemporal, is a reciprocal notion to discontinuity. Spatially, an edge that leads into a discontinuity is relatable to another when the two can be connected with no discontinuity in between. Just as the presence of a corner or very sharp curve defines a discontinuity in step 1 above, its absence defines relatability.

An additional condition defines relatability of edges. We will call it the monotonicity constraint. Intuitively, it requires that the connection must progress continuously from one edge to the other. The connection cannot extend outward and then return, or double back on itself, etc. Without this constraint, any two edges would be relatable, as Fig. 22 illustrates.

The notion of relatability, incorporating the monotonicity constraint, may be formally expressed in terms of two conditions.[8] Consider two surface edges, E_1 and E_2, and their linear extensions, s_1 and s_2. First, s_1

[8] We thank J. Edward Skeath for contributing many of the ideas in this section and for writing Appendix 2.

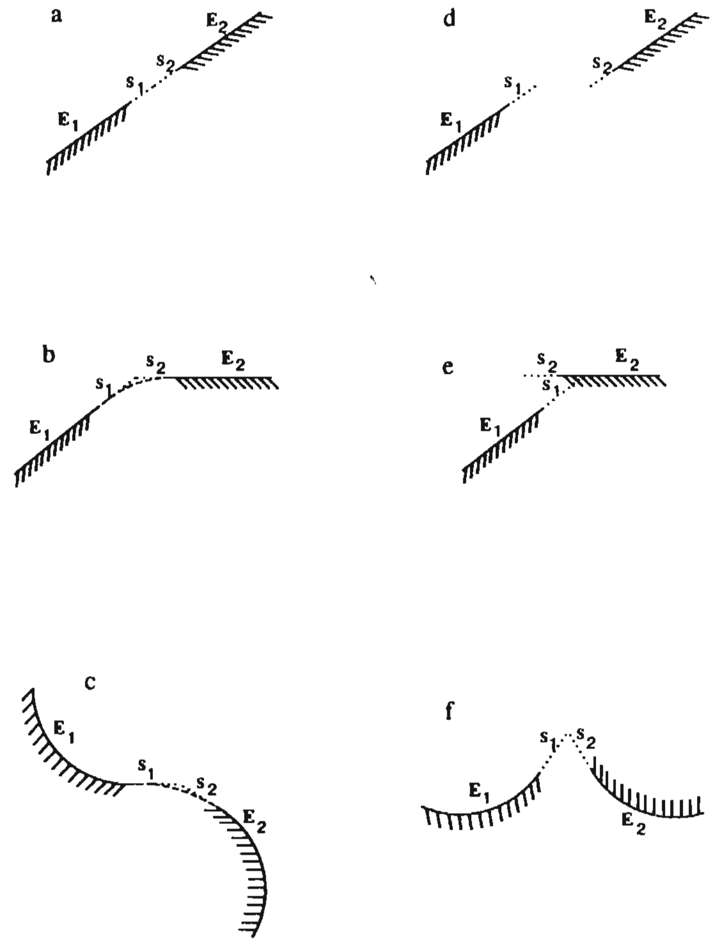


FIG. 23. Relatable (a, b, and c) and nonrelatable (d, e, and f) edges (see text).

must intersect s_2. Second, their angle of intersection must be obtuse or 90°. Figure 23 illustrates some relatable and non-relatable edges.

In the case of parallel edges, these criteria lead to different outcomes depending on whether the edges are aligned or not. For collinear edges, the extensions of the edges meet and are thus relatable. Misaligned parallel edges, on the other hand, do not have intersecting extensions. There is a small range of tolerance at the boundaries given by these criteria. In each case, we would expect the pick-up of edge relations such as misalignment to be subject to thresholds. The exact ranges of tolerance, and the drop-off of interpolation within them, have not been fully determined. However, the ranges appear to be narrow. For example, in the experiment described above on the identity of partly occluded and illusory figures, unit formation became marginal or impossible beyond about 15 min of visual angle of misalignment. This threshold estimate is well above Vernier acuity thresholds, but on the same order as errors in processing collinearity in the Poggendorf illusion (Robinson, 1972).

Except for the case of misaligned parallel edges, the conditions given can be expressed analytically in the following useful form. Considering again E_1 and E_2 as above, let R and r be perpendiculars to the end points of the two edges, assigned so that $R \geq r$, and let φ be the angle of intersection of R and r (see Fig. 24). Edges are relatable if and only if

$$0 \leq R \cos \varphi \leq r. \tag{3}$$

According to these criteria, relatability is blocked when the extension of one edge intersects the other before the end of the latter edge (since $R \cos \varphi$ exceeds r); this would be the case, for example, in Fig. 23e. For the edges shown in Fig. 23f, relatability fails because $\cos \varphi < 0$.

It can be shown that when two edges meet the relatability criteria, a first-order continuous curve can be fit between them, tangent to the end points at both edges. A proof is given in Appendix B. The curve described in the proof derives naturally from the construction in Fig. 24. There is an interesting sense in which it may be a minimum curvature connection between the two edges (see Appendix B). Whether this curve predicts the actual perceived form of interpolated edges remains to be investigated.

The criteria stated allow the possibility of edge formation in the absence of closed figures. Figure 25 gives an example.

The importance of the relatability criteria is easy to see. As noted

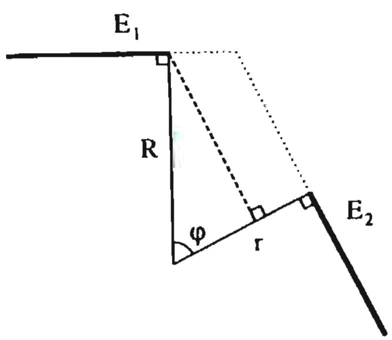


FIG. 24. A construction defining relatability. Two surface edges are relatable if and only if $0 \leq R \cos \varphi \leq r$, where R and r are perpendiculars to the ends of the edges, assigned so that $R \geq r$. The magnitude of r from its intersection with R to its intersection with the dashed line is $R \cos \varphi$ in this case (see text).

FIG. 25. Edge formation without area enclosure.

above, discontinuities in projected edges constitute an important invariant related to object perception. In an important respect, however, it is an invariant relationship running in the wrong direction. The geometry of visual perception ensures that every case of partial occlusion will generate first-order discontinuities in the projections of objects. The visual system's task, however, is the converse: to use the projections of objects to detect partial occlusion. However, not all first-order discontinuities arise from occlusion. Some unitary objects have sharp corners. How can the visual system determine which discontinuities in the optic array arise from occlusion and which arise from abrupt changes in the outer boundaries of objects? The relatability criteria provide the answer. When the edges leading into a discontinuity are relatable to others, the discontinuity is classified as arising from occlusion. This occurs, as we have seen, even when it entails the perception of boundaries within homogeneous areas. A useful way of summarizing the effect of the relatability criteria is that the visual system minimizes the discontinuities in the optic array which must be ascribed to the boundaries of objects in the world.

Evidence for relatability. The relatability criteria are consistent with most existing data on unit formation but need to be subjected to formal tests. Above we described data indicating that, consistent with relatability requirements, relatively small misalignments of parallel edges block unit formation. Further studies using equivalent occlusion and illusory figure displays are currently being carried out to test comprehensively other types of violations of the relatability criteria.

Two recent studies may illustrate the usefulness of the relatability notion. In one experiment, spatial relatability was tested in perception of partly occluded figures given over time (Kellman, Power, & Shipley, 1989). An anorthoscopic perception method was used to minimize information available at any moment. Specifically, two areas separated by a gap were revealed through two 76 by 4.7-mm (visual angle: 4.1° by 15′) slits separated vertically by 6.4 cm (3.4°) and misaligned horizontally by 3.8 cm (2.0°) in a moving occluding surface (see Fig. 26a). The middle area of the displays was thus always occluded. Relatability between edges of the two areas was varied. Five displays had relatable edges on both sides; four had non-relatable edges, and one display had relatable edges on one

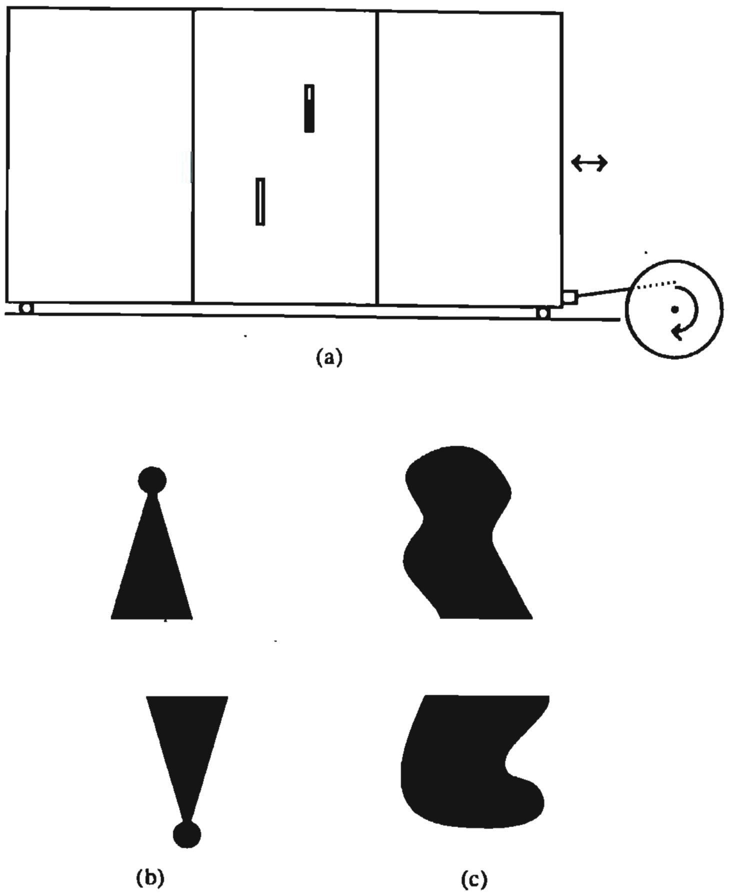


FIG. 26. (a) Anorthoscopic presentation apparatus. Stimuli with relatable (c) and (b) nonrelatable edges were presented.

side but not on the other. Figures 26b and 26c illustrate figures with nonrelatable (b) and relatable (c) edges. Violations of relatability consisted of three cases where $R \cos \varphi > r$ and one case where $R \cos \varphi < 0$. The major dependent variable was a forced choice by subjects as to whether one object or two was present behind the occluder in each display. For the displays predicted by relatability to be seen as one object, an average of 20.6 subjects (out of a possible 24) reported one object. (The fewest reports of one object for displays in this category was 15.) For displays predicted by relatability to be disconnected, an average of 19.8 subjects reported two objects. (The fewest reports of two objects for displays in this category was 18.) The display meeting the relatability

criteria on one side but not the other was reported as one object by 13 subjects and two objects by 11 subjects.[9] This experiment, although employing a small set of displays, suggests that relatability predicts unit formation under occlusion, even where the physically specified edges are given only over time.

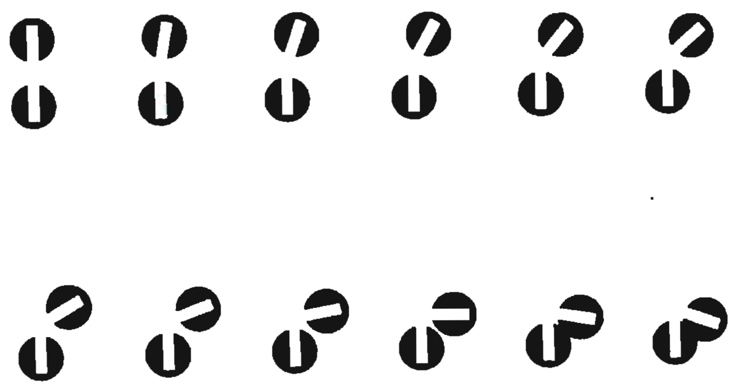
The occlusion experiment included little test of the hypothesized lower bound on relatability, i.e., $R \cos \varphi \geq 0$. Experiments in progress in a different context have begun to examine whether edges whose relative orientations are less than 90° fail to support interpolation. We have obtained preliminary evidence with illusory contour displays. Subjects were tested in two different ways. In one test, subjects were familiarized with the illusory contour phenomenon and shown a set of illusory contour displays differing only in the orientations of the physically specified edges (see Fig. 27a). Subjects were asked to check off all of the displays on the page in which they saw illusory contours. Figure 28 shows preliminary data from 15 subjects. There was only one report of an illusory contour for the two displays with orientation differences less than 90°. Similar data were obtained using a second method in which an illusory "string" was created in which separate segments required interpolation between edges of varying relative orientations. (An example of such a string is given in Fig. 27b.)

These preliminary data suggest that the relatability criteria are plausible, both in including cases where interpolation occurs and excluding cases where it does not. These data, and others, also suggest that strength of boundary interpolation may vary with orientation among edge pairs that meet the relatability criteria. There is some indication that collinear edges are strongest, and interpolative strength declines with deviations from collinearity. This possibility is consistent with proposals by Rock and Anson (1979), who noted that straight illusory contours were more frequently reported by subjects, and Grossberg and Mingolla (1985a), who suggested that the strength of boundary formation should be greatest between like-oriented edges. At the other end of the range, whether there is a cutoff below which relatability does not occur, or a very steep, but continuous, drop off in interpolative strength around 90°, is hard to judge from these data.

Three-Dimensional Relatability

Although most demonstrations of amodal completion and illusory con-

[9] The experiment actually tested unit formation at two different speeds of the moving occluder. Data given here are from the slower of the two speeds, since higher speeds in anorthoscopic perception tasks have been reported to induce some figural distortion. Our data showed only minor variation with speed, however.



(a)

(b)

FIG. 27. (a) Systematic variation of the relation between inducing edges from 180 to 70° in 10° increments. (b) An illusory string with variations in the relative orientation of inducing edges ranging from 180 to 70° (see text).

tours have produced edges that lie in a frontoparallel plane, this need not be the case. Three exceptions are a stereoscopic illusory figure developed by Gregory and Harris (1974), the subjective Necker cube of Bradley and Petry (1977), shown above in Fig. 11c, and several illusory figures examples discussed by Brady and Grimson (1981). Such observations suggest that relatability applies across all three spatial dimensions. It is clear from projective geometry that collinear edges and smooth curves in the world

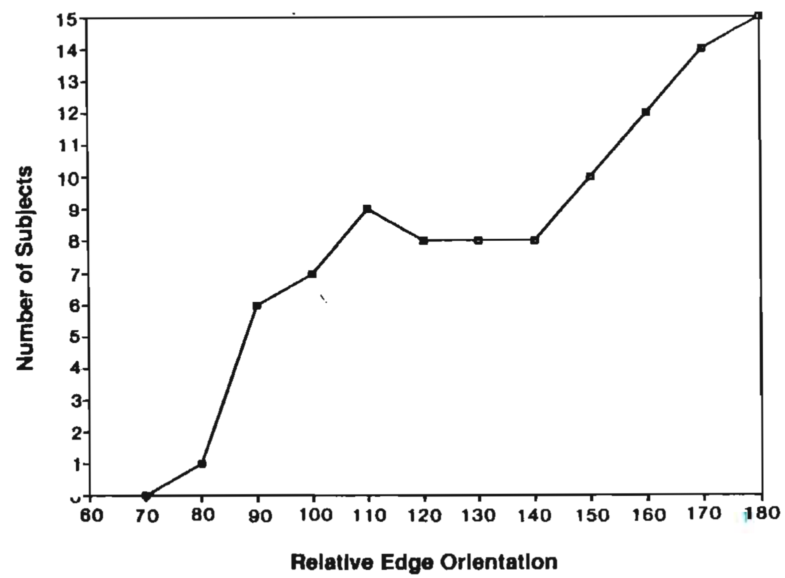


FIG. 28. Number of subjects (out of 15) reporting a subjective figure for each of the displays in Fig. 27a.

ordinarily project to collinear edges and smooth curves at the retina. The relatability criteria should thus apply appropriately to 3-D objects despite varying orientations to the viewer.

When depth information indicates that physically specified edges are oriented outside of the plane, then interpolated edges and surfaces should also lie outside of the plane. The stereo pair in Fig. 29 illustrates the effect. When the left and right views are presented to the left and right eyes, respectively, a ring tilted in depth is seen, with a curved illusory surface arching out of the plane on the right, in front of the surrounding surface, and a partly occluded curved portion of the ring, behind the surround, on the left. If the same two views are reversed, so that the left eye gets the right view and vice versa, the amodally completed and illusory figure sides of the ring reverse. Besides showing that the relatability notion applies in 3-D space, this type of display gives another illustration that amodal completion and illusory contours derive from one process: depth information determines whether interpolated edges lie in front of or behind other surfaces in the array.

A further implication of the 3-D relatability notion is that collinear (or smoothly curving) edges at the retina might not always be relatable. If two retinally collinear edges, for example, come from real objects at sufficiently differing depths or orientations, no smooth, monotonic connection

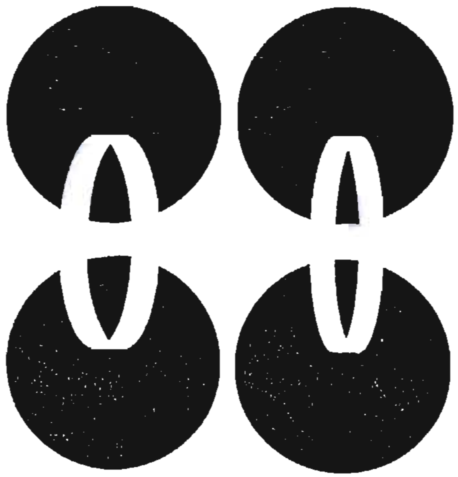


FIG. 29. Three-dimensional relatability. This stereo pair produces perception of a ring oriented at about 80° out of the picture plane, ordinarily seen as a three-dimensional illusory contour on one side and as partly occluded by the surface of the page on the other. Reversing the views given to the left and right eyes reverses the "amodal" and "modal" sides of the object. The reader may be able to obtain the stereoscopic effect by crossing or diverging their eyes.

between them in 3-D space may be possible. An example is shown in Fig. 30. Despite the collinearity of the edges, no illusory contours are seen in this display. We are currently investigating the question of 3-D relatability more formally, using stereoscopic displays, to determine the influence of depth separations on relatability.

Spatiotemporal Relatability

The conditions governing spatiotemporal relatability have not been thoroughly investigated. Thus, our proposed definition is conjectural. Spatiotemporal relatability should include at least those cases in which successive spatiotemporal discontinuities could be caused by a unitary, rigid object translating at a constant velocity or rotating at a constant

FIG. 30. Nonrelatability in three-dimensional space. Despite collinearity of the inducing elements' edges in two dimensions, three-dimensional orientation information appears to block illusory figure perception.

angular velocity (Kellman & Cohen, 1984; Kellman & Loukides, 1987). It is likely that the actual class of events constituting relatability will turn out to be broader. For example, a requirement of rigidity is probably too limiting here as it is in other cases of perception of structure from motion (Todd, 1982). Kinematic unit formation in illusory figure or occlusion cases may be possible with certain jointed or elastic objects, but these possibilities have not been tested. Likewise, lawful motion patterns other than constant velocity, such as simple harmonic motion or constant acceleration, might support unit formation.

3. A New Unit Is Formed When Connected Edges Enclose an Area

The optically specified edges, together with their extensions given by relatability, form a unit when they completely enclose an area in the optic array. Some test for boundary closure is needed. A simple idea is that, for an enclosed area, following along the physically specified and interpolated boundaries, one arrives back at the starting point. Highly complex areas, however, may present additional problems (Ullman, 1984), which we do not take up here. An enclosed area will normally appear as an object, but depending on depth information relating adjacent surfaces, it could appear as a hole of the same shape. (This issue of perceptual boundary assignment is taken up in the next section.) A related issue is how the system handles competition. What happens when an edge is relatable to more than one other edge, and what happens when a single area falls within two or more potential units? These issues require further study.

4. Units Formed Are Assigned Positions in Depth Based on Available Depth Information

The depth placement of units is not intrinsically part of the unit formation process. It is governed by available depth information about the surfaces in the array. The outcome of depth placement determines the appearance of formed units as "modal" or "amodal" in the terminology of Michotte et al. (1964). In Fig. 13a the triangular object appears behind the other object, because there is a surface color change between its two visible areas. This is the depth cue of interposition, which has an extremely interesting relation to the unit formation process (see Section VI). In Fig. 13b, there is no such color change across the boundaries of the central object. As noted above, the figure can be seen either in front of or behind adjacent surfaces. The dominant impression (illusory figure on top) is probably due to some weak depth cues (Rubin, 1915). Since the dark areas are enclosed by a larger surround, they tend to be seen as in front of the surround, rather than as holes in it. The completion of the central boundary of each dark area traverses an area of color change (belonging to the triangular figure), so the triangular figure lies atop the black areas by interposition. Therefore, the central figure must also lie atop the surrounding white surface. Note that this depth ordering is fully determined only when the black areas are seen as figures; when they are seen as holes, the triangular unit may be either in front or behind the surrounding white surface.

Especially instructive is the spontaneously splitting area. On sustained viewing the depth relations between the two units reverse. This outcome occurs because there is no depth information specifying the relation between the two objects. As argued above, this class of display is rather compelling in suggesting that the "amodal" vs "modal" difference in object perception has to do not with object formation, but depth ordering.

Perceptual Boundary Assignment

An important question is when in the process of object perception are contours assigned as one-sided boundaries. We currently believe that boundary assignment is dependent on depth placement. Where depth information is unambiguous, such as when stereoscopic or kinematic differences among surfaces are given, parts of the array are probably assigned spatial positions in advance of the unit formation process. Boundary assignments follow directly in that the nearer surface "owns" the boundary; the further surface, adjacent in the projection, is not bounded. Such early depth assignment can block relatability, i.e., when edges are collinear in the projection but differ in depth. When depth information is weak (as in pictorial displays), the depth placement of new units, as well as the assignment of closed boundaries as delineating figures or holes, may be bistable, as we have seen. It seems that in ordinary visual environments, the adequacy of relative depth information accounts for the lack of ambiguity in boundary assignment (Gibson, 1966); whereas, in pictorial contexts, the lack of adequate depth information makes possible reversals of boundary assignment.

VI. RELATED THEORETICAL ISSUES

In this section we consider several issues that, while not central to our theoretical framework, involve other pertinent aspects of the unit formation process.

Spatial and Temporal Range

The unit formation process is presumably limited in what sorts of spatial and temporal gaps it can bridge. There are almost no data on the effects of varying temporal separation. Some research has sought to determine how spatial gaps affect illusory contour formation. Dumais and Bradley (1976) varied both real and retinal sizes of illusory figures and reported a clear effect of retinal size. (Best edge clarity was found for

edges around 1.2° of visual angle in retinal extent; larger separations usually gave lower ratings.) Petry, Harbeck, Conway, and Levey (1983) found that subjects' clarity ratings of an illusory figure increased as the gap between inducing elements was decreased. Unfortunately, gap extent was confounded with size of the inducing elements in one experiment and with number of inducing elements in the other experiment (size and number of inducing elements were varied without changing the size of the illusory figure). Little is known about the interactions between the extents of specified edges and the gaps between them. In a recent experiment (Shipley, 1988; Shipley & Kellman, 1988), subjects gave magnitude estimations of illusory edge clarity as a function of various inducing element sizes and separations. Both distance between inducing elements and size of inducing elements at the four corners of a square were varied (crossing four gap sizes with three inducing element sizes gave 12 displays). The results are shown in Fig. 31. Both the extent of the gap and the extent of the luminance specified contour affected the clarity of illusory edges.

In a program of research that may have a number of connections to the present work, Gillam (Gillam, 1972, 1981; Gillam & Grant, 1984; Gillam & McGrath, 1979) investigated the conditions under which separate line segments rotating in depth appear to rotate in the same direction (as if connected) or not. Her data on spatial separation (Gillam, 1981) led her to

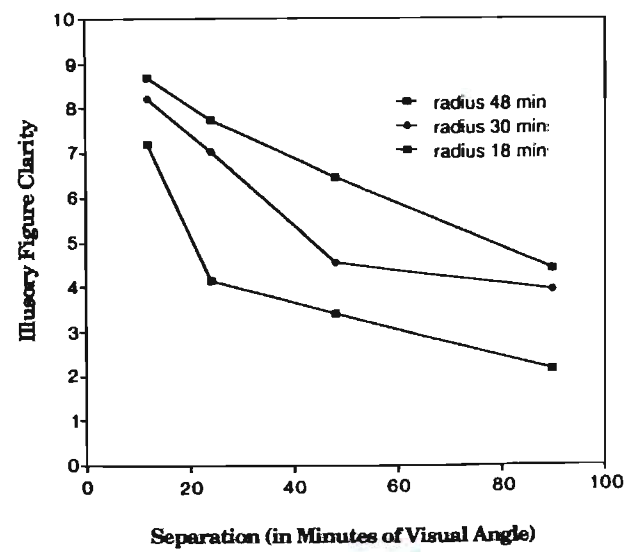


FIG. 31. Mean illusory contour clarity ratings as a function of inducing element separation and size (radii in minutes of visual angle) ($n = 20$).

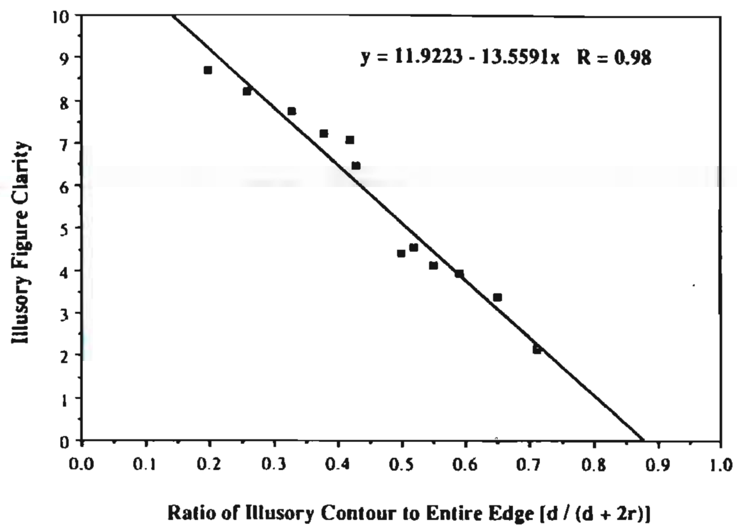


FIG. 32. Mean illusory contour clarity ratings as a function of the ratio of gap size (d) to total edge length. (Edge length ($d + 2r$) equals twice the inducing element radius (r) plus gap size (d). The solid line represents the best-fitting regression line, with parameters given by the equation in the figure.

propose that the ratio of edge length to gap size governed perceived coherence. Both increasing the length of specified edges and decreasing the size of the gap between them increase coherence. An advantage of the ratio hypothesis is that ratios are invariant with viewing distance; thus, configurations that appear unified at one distance will remain so at another.

Examination of our data suggests that the ratio hypothesis describes the effects of spatial separation on contour strength here as well. Figure 32 replots the data of Fig. 31 with illusory figure clarity shown as a function of the ratio of gap extent to the entire gap (luminance-specified parts plus the gap). Equivalent ratios give approximately equivalent clarity ratings. Within the range of ratios used in this study, illusory figure clarity seems to be a linear function of the ratio of gap to gap plus specified edge length.[10] Grossberg (1987) has also described cooperative interactions between edges in a way that is compatible with the ratio hypothesis.

Whether the ratio of gap extent to illusory figure extent is the only

[10] Moreover, other parallels may exist. Misalignment of edges may have similar effects in determining rotation coherence as in determining illusory edges and unity under occlusion. The possibility that unit formation in the rotation case may be partly or fully explained in the same framework as partly occluded objects and illusory figures is currently under study.

variable relevant to issues of spatial extent and unit formation is not clear. According to the ratio hypothesis, a retinal size change that leaves the ratio of the gap to the entire edge intact should not diminish perceived edge clarity. Thus, the retinal size effect reported by Dumais and Bradley (1976) is inconsistent with the ratio hypothesis. However, our stimuli spanned a smaller range (1–3° for the sides of the illusory figures) than those of Dumais and Bradley (1.2–18.9°). Moreover, the differences due to retinal size found by Dumais and Bradley in the range of our stimuli were small. It is possible that the ratio hypothesis holds only within a certain range of retinal extent; perhaps it is restricted to foveal and parafoveal viewing.

Complete vs. Partial Relatability

A discontinuity in the visual field usually involves two (projectively) intersecting edges. A question not addressed so far is whether unit formation is stronger or more stable when both edges leading into a discontinuity are relatable to others. Specifically, what happens under conditions of partial relatability, when only one of the two edges is relatable, and forms part of a new unit? Is that unit perceptually weaker or less stable if the other edge at the discontinuity does not become part of some other unit?

Ecologically, cases of occlusion leading to partial relatability would seem to be fairly common. When an object terminates behind another object occluding it, relatability for the occluded object's edges may be lacking. As an example, consider Fig. 33. The illusory contours here result from the relatable horizontal edges. The vertical edges, however, are arranged so that they are not relatable to other edges. Displays of this type, but made of very thin lines, have been considered by Gillam (1987); her examples indicate that clear illusory contours can be obtained in the absence of collinearity of inducing lines across the gap. Although experimental data have not yet been obtained, there is also some suggestion that illusory contours are no stronger when collinearity is absent. The noncollinear displays were not designed to test the relatability notion proposed here, and they may contain some relatable edges. Our informal observations have been inconclusive as to whether complete relatability improves unit formation, and we are currently initiating a more systematic investigation of this issue.

The Depth Cue of Interposition

It is clear that some configurations give the impression that one object is in front of another, and that another object continues behind. This source of depth information is usually called "interposition." It has proven difficult, however, to formalize adequately the conditions under

FIG. 33. Partial relatability. Illusory contours occur between relatable horizontal edges here despite the absence of relatability between the other edges leading into the discontinuities.

which interposition occurs (Hochberg, 1971). Our theory may shed some new light on interposition. The clearest examples of interposition may depend on edge relatability. Figure 34 illustrates this claim. Consider the impression of one object going behind another in displays (a), (c), and (e). In (a) there is complete relatability. The edges forming each discontinuity (e.g., the right angle formed by a vertical edge between the grey and the black regions and a horizontal edge between the grey and white regions) are both relatable to other edges. The horizontal edges of the two grey regions are related to each other across a gap—the black region. The vertical edges between the black and grey regions are relatable, with no gap, to the vertical edges between the black and white regions. The impression of one object going behind another may be strongest in this case, where all of the edges leading into the relevant discontinuities are relatable to other edges. Figure 34b is a second example of complete relatability in which the boundary of the grey surface is relatable to itself across the gap. The displays in (c) and (d) do not have complete relatability and may give weaker impressions of one surface lying behind another. In (c), only the vertical edge between the black and grey regions continues through the discontinuities. The horizontal edges are not relatable to other edges. In (e), none of the edges leading into the discontinu-

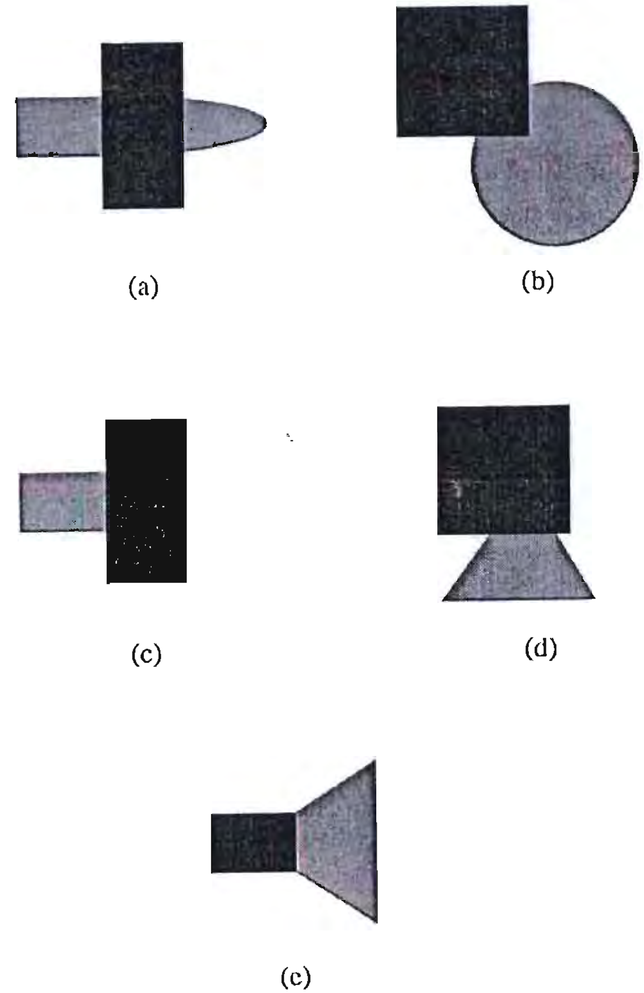


FIG. 34. Interposition and relatability. (a and b) Complete relatability. (c and d) Partial relatability. (e) No relatability (see text).

ities are relatable to other edges. (The oblique edges are not relatable to each other because $R \cos \varphi < 0$.) The two regions in this display evidence the weakest depth ordering and may appear adjacent to each other.

A plausible way to think of interposition is as a rule for determining boundary assignment (and hence depth ordering) given certain relatability conditions. The outcome depends on the relations of surface qualities when the projections of two objects overlap. A change in surface quality along the boundary of one object but not the other indicates that the former object is behind the latter. The shared boundary is assigned as belonging to that object, bounding it but not the other (further) object. In (a), for example, the black object is in front, because its surface quality is not interrupted where the projections of the two units overlap. Even when only one edge continues through a discontinuity, that edge is apparently assigned as in front of the other, although the impression may be weaker. In (c) and (d) the continuity of the edges between the black and grey regions with the edges between the white and black regions leads to those edges being assigned as boundaries of the black region. Thus, the grey areas in these displays are not bounded and continue behind. In these displays, the grey surface may appear behind the black one, despite the fact that the specific locations of the hidden boundaries are not given by the boundary interpolation process.

The fact that with partial relatability, the unbounded surface continues behind the other may be relevant to another issue. Sometimes figures whose edges are oriented at acute angles are reported to connect behind an occluding object. For example, the display in (d) may be described as containing a triangle. Such reports might indicate that the relatability criteria should be broadened to include acute angles and that sharp corners can be interpolated. However, such reports do not necessarily indicate relatability between edges oriented as acute angles. Rather, the facts that the visible portions are consistent with a triangle, and the remaining boundaries are not specified may be adequate for a display to be recognized as consistent with a triangle and reported as such. On this hypothesis, such reports would not be results of perceptual boundary interpolation, but might involve processes of recognition or reports based on partial information. One reason to think that this is the correct explanation is that the acute angles in (e) do not seem to give much impression of a whole triangle. If acute angles could be joined by the boundary interpolation process, (e) and (c) might be expected to give similar impressions in this regard.

The interposition rule seems to be one of a small set of rules governing the depth relations and appearances of projectively overlapping objects. Another is that color changes along *both* boundaries of two objects whose projections overlap are necessary for apparent transparency to be perceived. Table 1 and Fig. 35 set out some of the color change rules that seem to govern the perceptual outcomes after units are formed.

The determination of depth ordering by surface color relations deserves additional comment. The unit formation process that we have described as color blind, in that similarity of surface color plays no role in determining whether spatially separated projections will be perceived as a unit. However, as we have seen, surface color relationships seem to be crucial for the depth cue of interposition, which may determine the depth ordering of units formed. Moreover, surface quality may be central in a surface completion process that operates within bounded areas (see below).

TABLE 1

Surface Quality Relations Determining Apparent Depth Relations among Projectively Intersecting Objects

Phenomenal appearance of parallelogram	(a) Occlusion case	(b) Illusory figure case
Partly occluded	Different: 1, 2, 3 4 = 3	Different: 1, 2, 3
In front	Different: 1, 2, 3 4 = 1	Different: 1, 2 3 = 1
Transparent[a]	Different: 1, 2, 3, 4	Different: 1, 2, 3
Spontaneously splitting (Ambiguous depth)	Different: 1, 2 3, 4 = 1	—

Note. The table refers to diagrams in Fig. 35. "Different" means that the numbered areas have different surface qualities (achromatic and/or chromatic color, texture). Identical surface quality is indicated by the equality symbol.

[a] The appearance of the parallelogram as transparent or as seen through a transparent object depends not only on the surface quality differences but also on particular relations among the achromatic and/or chromatic colors in the array (Metelli, 1974; Ware, 1980).

VII. RELATIONS TO OTHER APPROACHES

In this section we discuss the relations between discontinuity theory and other theoretical notions that have addressed one or more unit formation phenomena.

Relation to Gestalt Psychology

The discontinuity theory of object perception proposed here is meant to formalize and replace the familiar lists of Gestalt principles, at least as they apply to object perception. (Whether the theory has any interesting relation to grouping phenomena, e.g., involving arrays of dots, is not clear, and such phenomena are not treated here.) In this section, we make clear the ways in which our framework incorporates, refines or supplants the best-known principles of Gestalt theory.

Good Form

The Gestalt psychologists argued that objects are perceived so as to have the simplest or most regular shapes. A variety of refinements of this basic theme have been proposed including information-theoretic approaches (Attneave & Arnoult, 1956), the "minimum principle" (Hochberg & Brooks, 1960), and coding theory (Buffart et al., 1981). As noted above, a major difficulty with such approaches is that they seem to require consideration of a (potentially large) variety of possible outcomes in

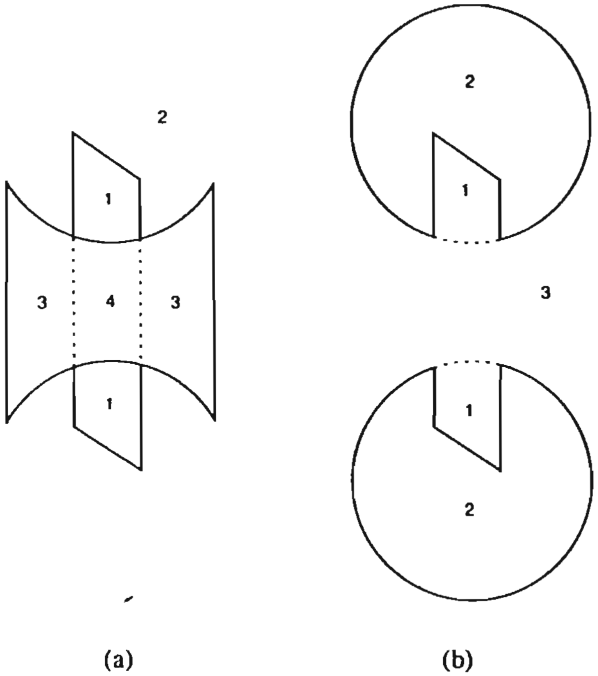


FIG. 35. Surface quality relations determining apparent depth ordering among projectively intersecting objects (see Table 1).

any given case. Our approach contrasts with these in that it is not "outcome-driven." The initiating conditions for unit formation are local discontinuities. The relatability conditions, while they demand the integration of information across regions of the visual field, would not seem to be computationally unwieldy. The overall form of objects is not a causal factor in perceptual outcomes.

Global simplicity, however, is a *consequence* of the principles set out here. The formation of units has the consequences that some discontinuities in the optic array are not assigned as parts of the boundaries of objects in the world. In fact, the process has the effect of minimizing the number of contour discontinuities that must be assigned to objects' boundaries. An interesting case is shown in Fig. 36. Figure 36a is a frequently used illusory figure inducing element. It has often been suggested that illusory figures are perceived because the visual system attempts to eliminate such figures with gaps, positing occlusion to turn them into more regular figures. When three such elements are arrayed in the Kanizsa triangle, for instance, perception of a central triangle allows the

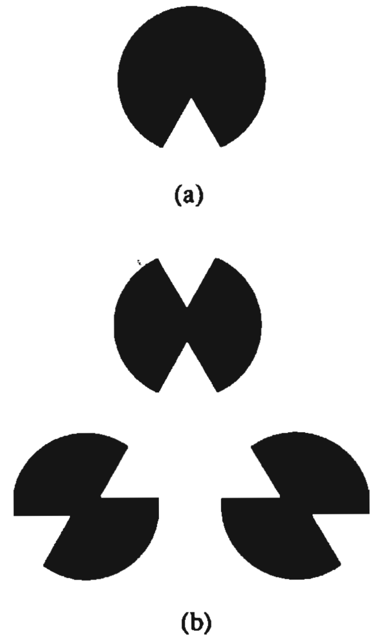


FIG. 36. (a) A single illusory contour-inducing element. (b) Illusory contour display in which a clear central figure is seen, despite the resultant reduction in symmetry of each inducing element.

inducing elements to be seen as partly occluded, but complete, circles. In our theory, the outcome is the same, but simplicity of the outcome is not a cause. Rather, local discontinuities initiate the phenomenon, and the relatability of edges gives both the illusory figure as well as the complete circles. The local nature of the phenomenon is shown in (b), where a robust illusory figure is seen. Instead of figural perception being caused by a perceptual aversion to the circular element with a single gap, here the notorious circle with a gap is created by the unit formation process! Notice that each inducing element would have greater symmetry if the illusory figure were not formed. In our opinion, simplicity and regularity of form are outcomes, not causes, of the unit formation process. If so, unit formation illustrates that global outcomes in perception may have local causes (Hochberg, 1978; Marr, 1982).

Good Continuation

Our theory might fairly be considered a formalization of the Gestalt notion of good continuation of contours, proposed by Wertheimer (1923), and applied to occlusion cases by Michotte et al. (1964). Kanizsa (1979) has emphasized contour smoothness in both amodal completion and illusory figure perception. Interpretations of the good continuation notion have been diffuse, however, involving a variety of stimulus attributes, and lacking formal definition or quantification. Our notions of discontinuity and relatability suggest that a more precise characterization is possible. These notions seem to be compatible with previous data on unit formation as well as with the results of experimental studies. Additional refinement is still needed; for example, perceptual thresholds for detecting discontinuities need to be determined. The basic importance of discontinuities in unit formation is already clear, however, and further empirical investigation of the discontinuity theory appears straightforward.

A final note on good continuation is that the current framework deals with discontinuities in the boundaries of *surfaces*. Most, if not all, demonstrations of good continuation have utilized line drawings. Such demonstrations are adequate for some purposes. In our view, however, line drawings have caused trouble in attempts to understand unit formation. Such drawings clearly differ in unit formation from arrays of surfaces that they are intended to represent; moreover, they introduce a number of curiosities as well as extraneous issues, e.g., symbolic interpretation. We take up some of these issues in Section VIII. For now, we simply note that our theory might be understood as a formalization of the good continuation notion, but only to the extent that that notion is taken to apply to the edges of extended surfaces.

Proximity. Some proximity notion no doubt plays a role in the process of unit formation among spatially separated visible areas As discussed earlier, it appears that within foveal and parafoveal viewing, the strength of unit formation depends on the ratio of the physically specified to unspecified edges of an object. Over wider ranges, the relevant proximity notion may be a retinal one, with decreasing strength for gaps of larger visual angles. Analogously, we believe that some temporal, or spatiotemporal, proximity metric is probably involved in unit formation over time, but this has not yet been investigated.

Similarity. Similarity of surface quality plays no role in the process that interpolates object boundaries. It does play a role in the depth placement of formed units, as we have seen. Moreover, there appears to be a surface completion process that complements the object formation process. This has been suggested previously in cases of modal completion (Grossberg & Mingolla, 1985a; Krauskopf, 1963; Walls, 1954), but its role in occlusion cases has not previously been observed. We take up this topic briefly in Section VIII below.

Other Perspectives

Besides the Gestalt tradition, our approach has interesting relations and

contrasts with other perspectives. These are examined below, organized by specific issues.

The identity of unit formation in occlusion and illusory figure cases. The unification of these phenomenologically diverse cases of unit formation is incompatible with some theories about phenomena in one domain or another. One example may be Grossberg and Mingolla's (1985a, 1987a) account of illusory contours. In this theory surface quality perception (brightness, color) plays a determining role in boundary perception. Although the "boundary contour" system determines the location of illusory edges, such edges will not be realized perceptually unless there are surface quality ("featural") differences across the boundary. In this regard, their approach is a successor to earlier brightness-based theories (Day & Jory, 1978; Jory & Day, 1979; Kennedy, 1978a). As Grossberg and Mingolla (1987a) put it: "In our theory the presence within the Feature Contour System of different filled-in featural signals on opposite sides of a boundary is necessary for sustaining visible figural form."

Recent research findings cast doubt on this claim (Kellman & Cohen, 1984; Kellman & Loukides, 1987; Parks, 1984; Prazdny, 1983). In studies reported by Kellman and Loukides (1987), subjects were given precise control over the brightness characteristics of illusory contour displays with oppositely contrasting inducing elements on a medium gray background (see Fig. 37). Subjects were instructed to try to make the edges of the central figure disappear and were also instructed to try to remove any difference in surface quality between the central area and the surround. All subjects reported success in matching the central and surrounding areas of the display; in contrast, subjects were unable to remove the central figure with clear edges. In short, clear boundaries can be seen in the complete absence of "featural differences." Other phenomena also indicate the irrelevance of featural differences as causes of illusory figures, including kinetic illusory figures and illusory figures generated using kinetic specification of inducing elements. In these examples, figural boundaries are seen in the absence of surface quality differences; moreover, because of their dependence on continuous change, no account in terms of brightness-based edge perception seems feasible.[11]

[11] The finding that equiluminance of inducing figures and surround tends to prevent illusory figure perception (Brussell, Stober, & Brodinger, 1977; Gregory, 1977; Livingstone & Hubel, 1987) may be misconstrued to be at variance with these claims. It is important to distinguish between the claim that the process of visual interpolation is a consequence of brightness perception (which seems clearly false) and the claim that the inputs to the unit formation process must be specified in certain ways, such as by luminance differences (which seems probable). The data indicate that both luminance-specified and motion-specified inducing edges are adequate inputs to the unit formation process, but that chromatically specified ones may not be. In this context, it is interesting to note a recent

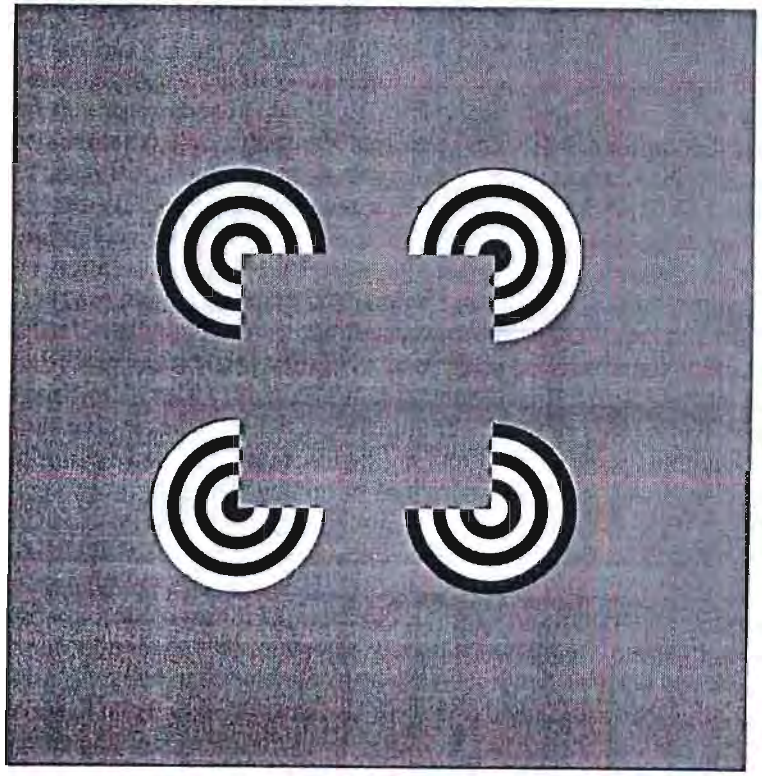


FIG. 37. Illusory figure without perceived surface quality differences across the illusory edge.

The most significant implication, however, of the claim that boundary perception depends on differences in surface quality is that the theory cannot readily be generalized to amodal completion. The perception of partly occluded objects, while accurate in determining occluded parts of boundaries, is clearly "amodal" in terms of surface qualities in the hidden areas (Michotte et al., 1964). Figure 38 shows an example in which unit formation operates but surface qualities of the figure in the occluded regions are not specified. Moreover, in terms of featural qualities in the *image*, in the central region, these belong to the occluding object, rather than the occluded one.

If, as we have suggested, a single process underlies modal and amodal

observation (Livingstone & Hubel, 1987) that arrays containing only chromatic color differences also fail to produce amodal completion. This observation is consistent with our hypothesis of identical mechanisms underlying modal and amodal cases.

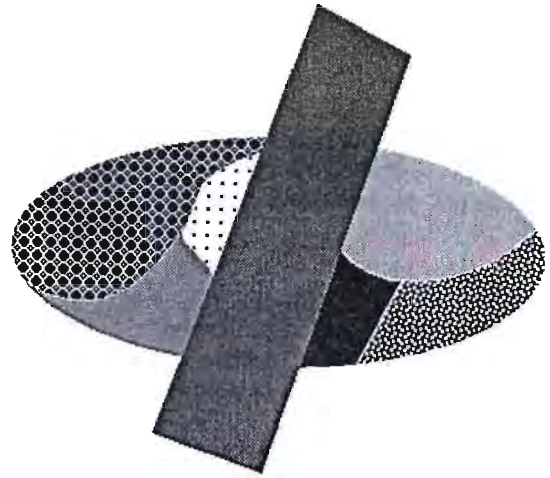


FIG. 38. Demonstration of perceived unity with indeterminate (amodal) surface quality (see text).

unit formation, "feature" or surface-quality-based theories cannot be adequate. In Grossberg and Mingolla's interesting approach to interpolative processes, there may be ways of modifying the assumption that only boundaries based on featural contrasts become visible. Perception of unity in occlusion cases and some aspects of illusory figure perception seem more related to processes that Grossberg and Mingolla have sought to describe in their "boundary contour" system.

First-order discontinuities as necessary conditions for unit formation. Parks speculated (1986, footnote 4) that abrupt changes in contour tend to characterize illusory figure displays, but that that abruptness has not been adequately specified. Coren's (1972) claim that certain boundary configurations function as implicit interposition cues is a clear precursor to our model. A significant difference is apparent, however. Although unit formation has an important relation to the depth cue of interposition, "implicit depth" or the recognition of figural gaps are not the bases of unit formation in our model. Discontinuities and relatability operate solely to form units, and depth information, including interposition, determines the positioning of perceived units. Brady and Grimson (1981) discuss a variety of types of discontinuities (luminance discontinuities, surface discontinuities, tangent discontinuities, etc.) in their consideration of surface perception.[12]

Implicitly, it might be argued that abrupt changes in boundary direction

[12] We thank D. Hoffman for calling our attention to the work of Brady & Grimson (1981).

will have unique consequences in Grossberg and Mingolla's (1985a,b) model. This is not completely clear, however. Grossberg and Mingolla indicate that in the vicinity of sharp corners, neurons selective for many orientations will be weakly activated. This would appear to be the case as well for curved edges. A process of "perceptual end cutting" is posited however, to avoid unwanted consequences of such arbitrary activation. Thus, at the end of a line, detectors of the same orientation at adjacent spatial locations are inhibited, while perpendicular ones are activated. With suitable choices of parameters, it appears to us that this perceptual end cutting will operate more strongly with abrupt corners than with curved edges. If so, a discontinuity theory and Grossberg and Mingolla's theory could make some similar predictions. However, the two approaches may be separable empirically on this point. Figure 39 shows an illusory figure display in which strong contours are normally seen. From our understanding of Grossberg and Mingolla's theory, there should be little or no activation along the illusory edges. The reasons are (1) the real

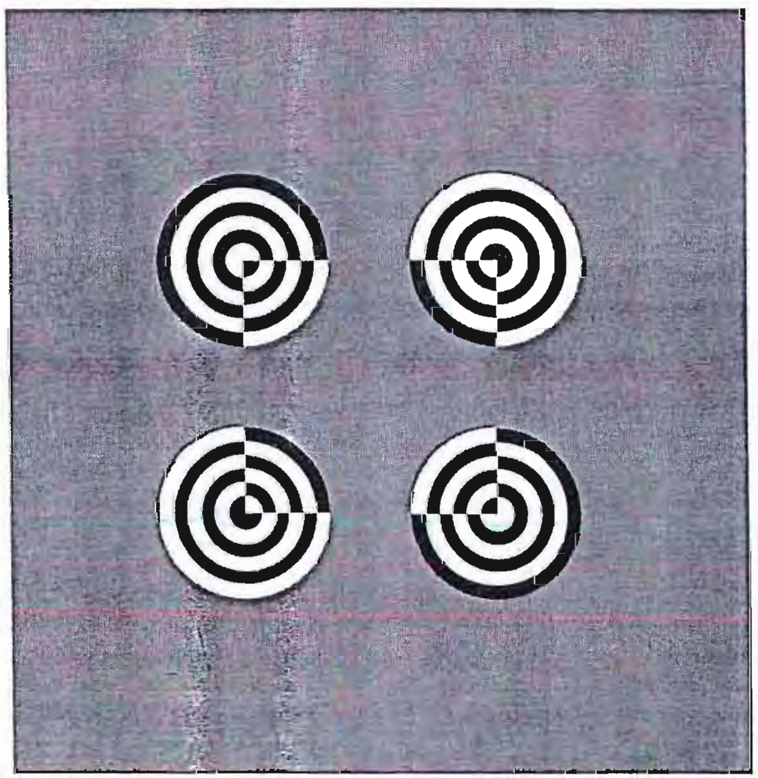
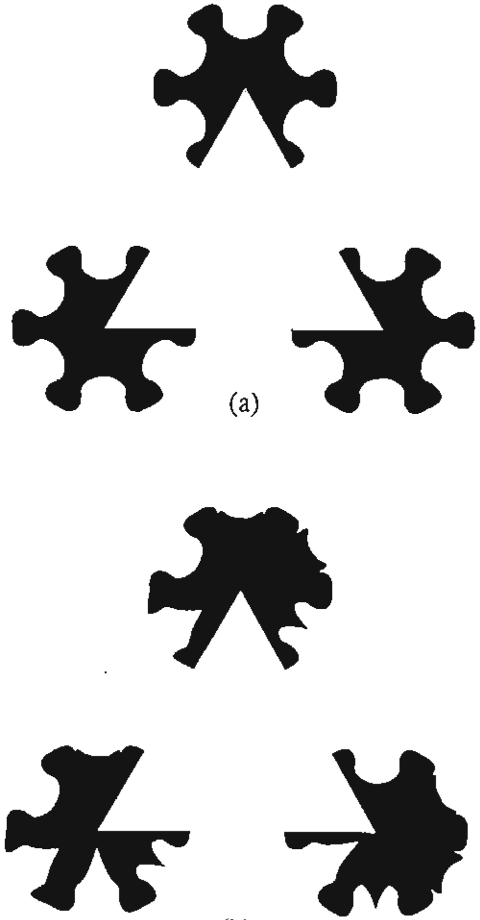


FIG. 39. Illusory contours formed without "perceptual end-cutting" (see text).

edges collinear with the illusory ones strongly inhibit their extension beyond the circles, because of end cutting, and (2) the curved boundary intersecting the line end produces no end-cutting because it does not end, according to the model. The latter fact follows from the insensitivity of boundary detectors to direction of contrast (Grossberg & Mingolla, 1985a). Whether this specific prediction is made by the Grossberg and Mingolla model depends on the values of a number of parameters which have taken on different values in their simulations (E. Mingolla, personal communication). Our discontinuity and relatability notions must predict the illusory figure seen here. (It was in fact developed as a prediction from the theory.)

The necessity of first-order discontinuities in surface boundaries does not seem to have been noticed previously in discussions of amodal completion. The coding theory proposed by Buffart et al. (1981) seeks to account for perceived unity and boundaries by simplicity criteria. As such, it makes many predictions in harmony with our model. (Recall that a consequence of our unit formation process is a minimization of the discontinuities in optical projections that are assigned as discontinuities in the boundaries of objects.) In general, coding theory requires for each discontinuity at least one, and perhaps two, new code elements (Buffart et al., 1981). However, where predictions overlap, our theory and coding theory make them for different reasons. Specifically, coding theory predictions depend on the overall symmetry and simplicity of perceptual outcomes. Contributors to symmetry and simplicity which do not involve our discontinuity and relatability criteria should provide useful test cases. This observation is tempered somewhat by the fact that descriptions of coding theory often present the general approach without making a clear commitment to a particular code. Nevertheless, it is possible to generate some predictions that would be sustained by almost any plausible code. One such prediction is that unit formation should tend to occur more strongly when the outcome would make figures more symmetrical than otherwise. Another is that overall symmetry of visible parts should lessen their ability to be combined with others in unit formation. Both of these predictions have been tested in recent experiments (Shipley, 1988; Shipley & Kellman, manuscript in preparation).

Subjects reported equally clear illusory figures in displays in which the inducing elements could be completed as symmetrical figures and in displays where such completion was impossible (for example Figs. 40a and 40b, respectively). Subjects also showed no reliable difference in the clarity rating of illusory figures seen in displays where the inducing elements contained multiple axes of symmetry prior to completion (for example, Fig. 41a) and displays in which the inducing elements had a single axis of symmetry (Fig. 41b).



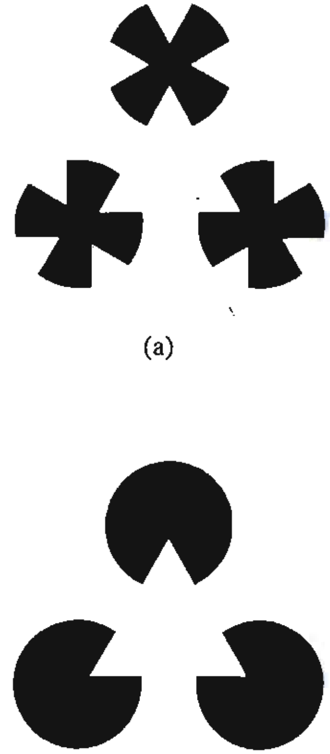
(a)

(b)

FIG. 40. Displays used in the symmetry experiments. (a) Inducing elements are potentially symmetric, i.e., they could be completed amodally as symmetric figures. (b) Inducing elements in this figure are not potentially symmetric and could not be completed amodally as symmetric figures.

Relatability criteria. As indicated above, the relatability criteria embody the main insight of the Gestalt principle of good continuation but the particular conditions governing relatability in our theory have not previously been proposed.

In an interesting paper, Ullman (1976) attempted to specify mathematically the forms of curved illusory contour edges. Like our criteria, his proposals preclude discontinuous connections between inducing edges. Ullman's paper was less concerned with the conditions under which unit



(a)

(b)

FIG. 41. Displays with (a) multiple and (b) single axes of symmetry (see text).

formation (or edge formation) occurs than with the particular perceived forms of connections when connections do occur. In complementary fashion, our theory indicates when unit formation occurs, but says little about the specific appearance of curved completions. Brady and Grimson (1981) discuss possible forms of smooth interpolated surfaces in three dimensions.

Grossberg and Mingolla (1985a,b, 1987a,b) propose that interpolation ("long-range oriented cooperation") of edges depends on "like-oriented masks that are approximately aligned across perceptual space" (1985a, p. 177). The notion of similarity of orientation contains some subtleties. Figure 42 shows three possible relations between illusory figure inducing areas of two different orientations. When the edges are placed one directly above the other (Fig. 42b), or when the lower edge is displaced to the left (Fig. 42a), no illusory figure is seen. Displacement to the right

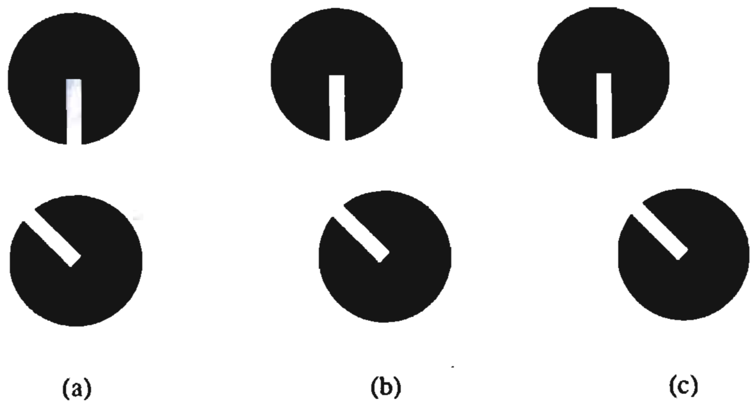


FIG. 42. Misalignments in opposite directions do not produce equivalent effects (see text).

does give an illusory figure. If orientation and alignment are taken to be separate notions, it is unclear why displacement in different directions should have different effects. In the detailed description of their model (1985b, Appendix 1), Grossberg and Mingolla handle this problem by defining orientation, not relative to the whole visual field, but relative to the cooperating edge. The relatability notion in our model shares and makes explicit this relative character of interpolation. Neither alignment nor orientation in the field can be considered separate determinants of interpolation. Further comparisons between our relatability criteria and the cooperative interaction proposed by Grossberg and Mingolla are difficult to make. The specifics of cooperation in the Grossberg and Mingolla model depend on several parameters which have not been fixed [notably the parameters P, R, T, r, and k in the spatial kernels F and G (1985b, p. 1970)]. Depending on the choices for these parameters, the equations for cooperative interactions could closely match our relatability criteria or be quite different.

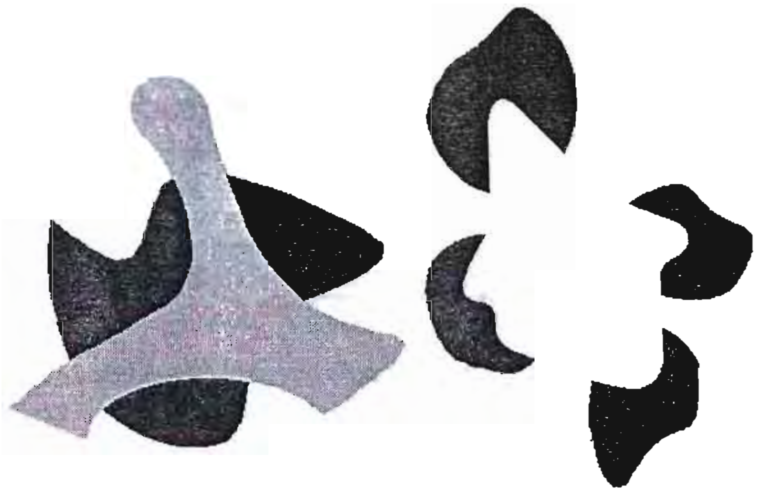
The sufficiency of discontinuities and relatability in unit formation. The explanation of unit formation in terms of discontinuity and relatability omits many other factors that have been suggested as playing some role in unit formation. Here we discuss two important ones, the importance of concavity in the shapes of illusory contour inducing elements and the importance of familiarity in perceiving illusory figures and partly occluded objects.

Convexity and concavity. It has often been argued that concavity of inducing elements is important to unit formation in illusory figures. Successful configurations are said to contain elements with "gaps" or "chunks taken out" (Brady & Grimson, 1981; Coren, 1972; Kanizsa, 1979; Rock & Anson, 1979). Concavity is also important to Hoffman and Richards' (1984) model for locating the natural parts of objects. One interesting consequence of our discontinuity and relatability criteria is that convex inducing elements are also predicted to support unit formation. Figure 43 shows an example of an illusory figure display with convex inducing elements. We believe that prior emphasis on concavity of inducing elements derived from the relative ease of enclosing areas with concave inducing elements. For example, in Fig. 43, the inducing elements lie within the illusory contour, giving a somewhat confusing appearance of figure or hole, although the contour is robust.

Familiarity. A number of investigators have suggested that familiarity plays a role in unit formation. Rock and Anson (1979) have asserted that familiarity of the partially specified figure is important to illusory figure perception, as is familiarity of the amodally completed inducing elements. Recently, Wallach and Slaughter (1988) suggested that reports of illusory figures reflect memories of familiar figures. Grossberg and Mingolla (1985a) also allow top-down influences to exert effects in their model.

The present framework accounts for unit formation in the various domains without invoking memory or familiarity effects. Unit formation should occur in any display which fulfills the discontinuity and relatability criteria, even if both the inducing elements and the unit formed by interpolation have unfamiliar, asymmetrical shapes. Such figures are readily generated. Figure 44 gives some examples, in occlusion (Fig. 44a) and illusory figure (Fig. 44b) cases. The paucity of such irregular illusory figures and partial occlusion displays in the literature perhaps reflects the prevalence of familiarity- and symmetry-based approaches. Alternatively, it may be the case that recognition of the relevant stimulus attributes (discontinuities and relatable edges) makes generation of such figures easier than before.

FIG. 43. Illusory figure from convex-inducing elements.



(a) (b)

FIG. 44. Unit formation does not depend on familiarity or regularity of inducing elements or units formed. (a) Partly occluded object. (b) Illusory figure.

VIII. UNSOLVED PROBLEMS

The present theoretical approach, we believe, provides a unified and novel account of many phenomena. In this section, we consider several phenomena that appear to pose challenges to the approach. Some of these phenomena may turn out to involve factors that properly fall outside of the theory of unit formation, while others may require extensions or modifications of some particulars of the theory.

Inputs Other than Surface Edges

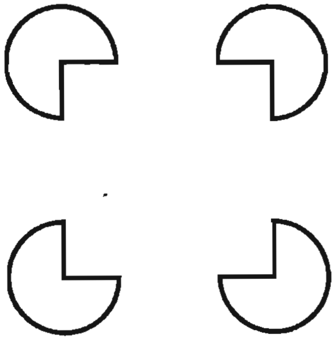
In our theory, the inputs to the unit formation process are the edges of surfaces. The discontinuities that initiate unit formation refer to abrupt changes of contour direction along such edges. There are some unit formation demonstrations that involve inputs other than these (or degenerate cases of these), specifically small dots and thin lines. Some aspects of the behavior of lines and dots seem to be treatable in our framework, by considering them as limiting cases of edges and of edge discontinuities respectively. As others have pointed out, however, lines and dots do not act in all respects as surface edges and corners, and may involve considerations unique to pictorial interpretation. In this section we survey these issues.

Dots. Edges can be interpolated between a physically specified edge and a dot. Minguzzi (1987) and Sambin (1987) give some examples for illusory contours. Two edges at acute angles which do not support interpolation alone can give rise to clear illusory contours meeting at a corner if a dot is placed at the intersection point of the extensions of the lines. Why does this phenomenon occur? It is possible that under some circumstances a dot functions as a discontinuity, i.e., that the mechanism which detects first derivative discontinuities is triggered by small dots as well. The consequent behavior of dots seems explicable on this hypothesis: Perceived connections between an edge and a dot are smooth and monotonic. When dots are placed in positions that would not allow such a connection, interpolation fails. Sambin (1987) gives examples of dots that do and do not relate to nearby surface edges.

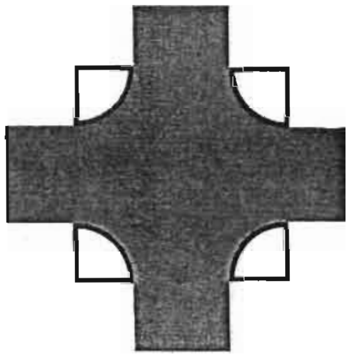
A technical point in applying the formal relatability criteria to dots is that dots have no edge orientation. We offer the following conjecture as to how relatability between a dot and a given surface edge may be determined. The dot may be assigned to the edge orientation that would result in a connection of constant curvature (if such a connection is possible) between the edge and the dot. Specifically, this means that orientation would be assigned so that $R = r$. It is a simple matter to determine when no such assignment is possible; the dot must be in the region whose boundaries extend from the edge's tangent $+/-45°$. This property of constant curvature, when only one edge tangent is given, is appealing in light of the relation of tangents (and perpendiculars) to curvature described in Appendix B. (The length of each perpendicular extending from an edge to the intersection of perpendiculars constitutes a radius of curvature that is basic to constructing the interpolated edge.) When only one oriented edge is present, there is only one radius of curvature to be considered. This proposal has not yet been tested.

Lines. Although there are some perceptual processes in which outlines are treated similarly to the edges of surfaces, there are also conspicuous differences for many perceptual concerns (see, e.g., Kanizsa, 1979; Kennedy, 1988; Sambin, 1987). Unit formation is one context in which the function of surface boundaries and lines is not equivalent. For example, it has been known for some time that outlines of illusory figure inducing elements (Fig. 45a) fail to produce illusory figures (Kanizsa, 1979; Sambin, 1987). The equivalence of illusory figures and partly occluded objects in our theory leads us to the novel prediction that outlines of the visible parts of partly occluded objects should similarly fail to produce unit formation as Fig. 45b shows; this prediction appears to be correct.

What characteristics of outlines cause their failure as inputs to the unit formation process? Outlines of figures can contain sharp changes in direction, and detecting the orientation of the edges leading into these dis-



(a)

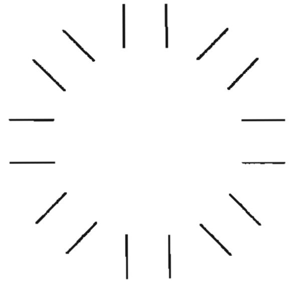


(b)

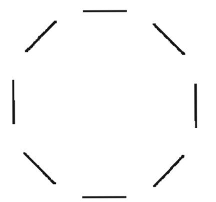
FIG. 45. Outline figures do not trigger unit formation. (a) Illusory figure case. (b) Occlusion case.

continuities would not seem to pose a problem. We do not yet have a fully satisfying account of the behavior of lines and their differences from surface boundaries, but we offer the following observations.

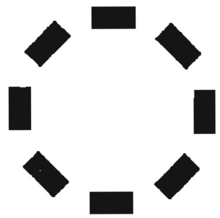
First, difference in the behavior of lines and surface edges seems to involve the "unextended" dimension of lines. Just as an abstract geometrical line is one-dimensional, so it may be that, perceptually, when a line's narrow dimension has less than some minimum extent, it fails to function as a surface in the unit formation process. Figures 46b and 46c illustrate this conjecture. In Fig. 46b, unlike Fig. 46a, the lines surrounding the central area do not give rise to an illusory figure in that central area. Figure 46c shows that increasing the thickness of the lines leads to perception of a central illusory figure. Figure 46d makes the same point by comparison with Fig. 46a. When the extended dimension of the lines in



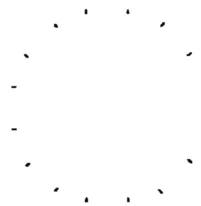
(a)



(b)



(c)



(d)

FIG. 46. Unidimensional functioning of lines. (a) Illusory contours are produced perpendicular to extended lines. (b) Illusory contours are not induced by the sides of lines. (c) Widening lines into extended surfaces allows the same edges as in (b) to induce illusory contours. (d) Reducing the extended lines in (a) to minimally extended segments eliminates illusory contour perception.

Fig. 46a are reduced to mere points, unit formation is blocked, although the terminations of the original lines in the central area remain unchanged.

A different, possibly complementary, hypothesis is that the behavior of lines along their narrow dimension results from their having two edges so close together. A line may be considered to be a very narrow surface with two parallel edges. There may simply be a rule of perceptual boundary assignment that when two edges of a homogeneous area are very close together the area must "own" both boundaries.[13] If such a rule applied, it could block relatability of one boundary to others when the resultant unit formation would lead to ownership of one boundary by an occluding

[13] This conjecture pertains to surface perception. The ability of line drawings to trigger object recognition may rely on a more abstract level of perceptual representation or cognitive skill.

surface. Thus, formation of a central figure in Fig. 46b would be blocked, although perception of the black lines as threads connecting behind the white surface should be (and appears to be) possible. The hypothesis of early boundary assignment in the case of two minimally separated edges is somewhat ad hoc, but it nevertheless has some ecological plausibility. It has the effect that configurations such as Fig. 46a cannot be seen as having a large occluded surface that appears only along a very thin (black) line outside a nearer, occluding surface. In ordinary viewing, occlusion of one object by another so that only a thin, constant width line of the rear surface reflects light to the observer would be highly improbable.

In other ways, lines do participate in edge formation. It is clear, for example, subjective contours can be readily induced orthogonal to extended lines (Gillam, 1987; Kanizsa, 1979; Kennedy, 1978b). Figure 46a gives an example. Consistent with discontinuity theory, the *shapes* of line ends are important for this effect. Kennedy (1988), Minguzzi (1987), and Sambin (1987) have shown that when line ends have clearly rounded tips, interpolation does not occur. Moreover, if the ends of lines are given detectable pointed shapes (as pencil points), interpolation is also blocked (Kennedy, 1988), as our relatability criteria would predict.

Some lines are so thin, however, that they do not have detectable orientations at the end. The behavior of these lines is problematic. In some cases, interpolation appears to proceed with a default orientation of the line end perpendicular to the extended dimension of such thin lines (or perpendicular to the tangent of the end for curved lines). This default orientation may be due to characteristics of edge detection processes needed to avoid indeterminacies at line ends (Grossberg & Mingolla, 1985a). The relatability criteria applied to this default orientation would predict that displacing the ends of lines so that relatability is violated should disrupt unit formation. This sometimes appears to be the case (Gregory, 1987). Examples can also be found, however, in which line ends appear to function more like dots. The rules governing interpolation where the ends of very thin lines are involved remain to be clarified.

Lines as constituents of perceived surfaces. A final interesting phenomenon involves groups of lines. In Fig. 47, an illusory edge may be seen connecting the discontinuities in the several lines. The relatability criteria would not seem to predict these edges, because it is not clear that any edges at the discontinuities are relatable to others, least of all in the direction that the illusory contour is seen. Another factor may be at work here. Closely spaced lines appear to be treated as surfaces, and the discontinuities mark the edge of intersection of two planes. If this is the case, then the perceived edge relates discontinuities along the edges of the planes. Further work is needed, however, to understand the conditions under which lines or other elements are grouped to form surfaces (cf., Gillam & Grant, 1984), as well as how these surfaces serve as inputs to the unit formation process we have described.

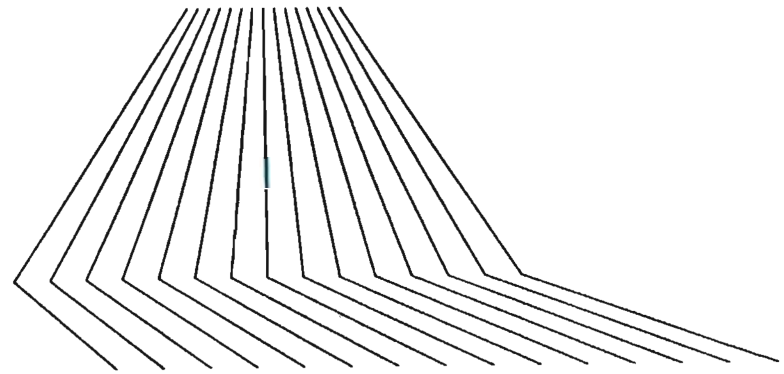


FIG. 47. The illusory contour probably depends on the grouping of lines into surfaces.

Perception of Unspecified Corners

Some partly hidden object displays are identified as containing squares, triangles, or other figures with unspecified edges meeting at corners. Figure 48a is ordinarily described as appearing to be composed of two squares. The interpolation of a sharp corner would not be predicted by discontinuity theory. Likewise, figures with edges oriented at significantly less than 90° should not give rise to edge interpolation at all; yet, occluded triangles may be reported as triangles.

As noted above, it is possible that reports of corners do not arise from the boundary interpolation process. When a sharp corner of a square or triangle is reported in occlusion cases, it may reflect recognition or report from partial information. In Fig. 48b, both alternatives may be consistent with the information given, but one is more readily reportable. When boundaries with indeterminate relations are not occluded, as in Fig. 48c, the outcome is vague, as has been reported previously in the case of edges oriented at 90° (Gerbino & Kanizsa, 1987; Ullman, 1976). Moreover, the experiment on acute angle relations and illusory figures described above suggested that boundary interpolation does not occur much beyond 90°. Further research is needed to determine whether interpolation of corners is a case where the boundary interpolation process actually differs for occlusion and illusory figure cases, or whether there is merely a differing response bias in cases of occluded and unoccluded boundary indeterminacies. One way of separating these possibilities experimentally would be to develop some technique to probe perceptual representations for specific boundary locations. If, for example, response bias accounts for a

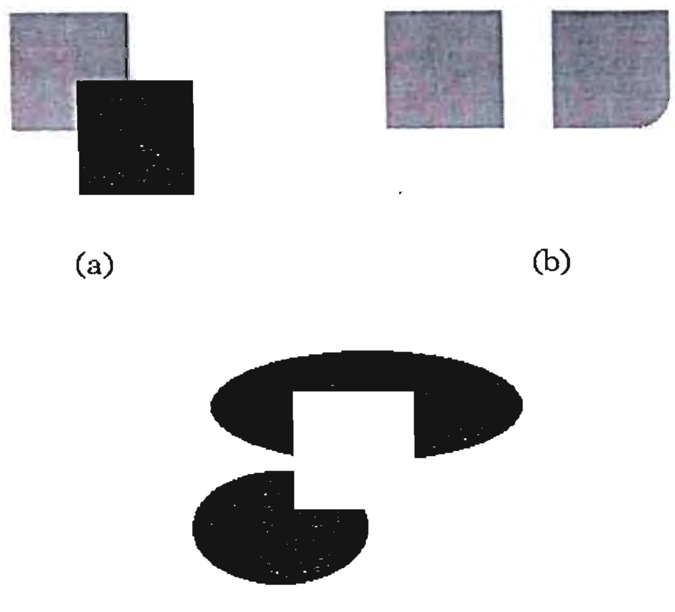


FIG. 48. (a) Occlusion display usually said to contain two squares. (b) Alternative completions of partly hidden surface in (a). (c) Illusory figure analog of (a) (see text).

report of a corner, its location might be indefinite compared to that of a truly interpolated boundary. A final complexity is that a surface completion process, separate from boundary interpolation, may be manifest somewhat differently in cases of partial occlusion and illusory figure perception (see below).

Competition

We do not yet know what rules apply when an edge is relatable to more than one other edge in the visual field. It is possible, for example, that in cases of competition the relation of minimum curvature predominates. This possibility would be likely if relatability turns out to vary quantitatively as a function of relative edge orientation. One might expect that extreme cases of multiply relatable edges would give rise to indeterminate or ambiguous perceptual outcomes.

Surface Completion: A Complementary Process?

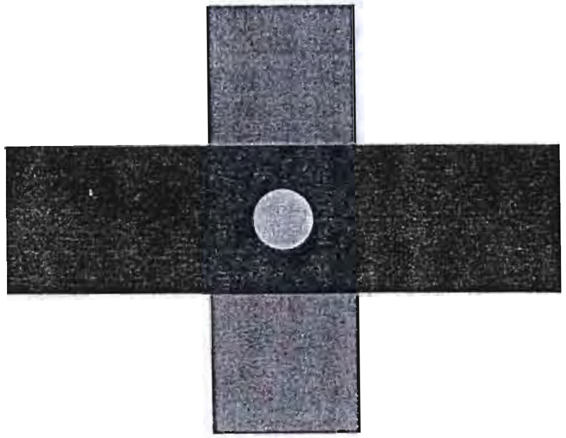
As we noted earlier, the process that determines the boundaries of objects is "color blind" in that differences in surface quality do not affect

unit formation. Although not involved in boundary determination, surface properties may play an important role in unit formation that is complementary to the determination of boundaries. There appears to be a surface completion process that can connect areas *within* the boundaries given by the boundary process. In Fig. 49a, a spot is seen lying atop a rectangle, unconnected to any other areas of the display. The circular area has no discontinuities along its boundary, and its boundaries should not connect to other areas. A similar circular area, however, of the background color *does* appear to be connected to another area in Fig. 49b: the circular boundary appears to be a hole through which part of the rectangle behind can be seen.

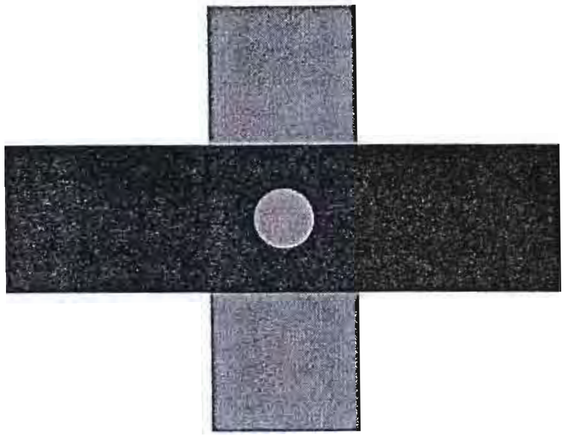
The difference between Figs. 49a and 49b suggests that surface completion occurs within boundaries given by the boundary completion process. Figure 49c presents this possibility most starkly. Identical circles that fall within or outside the projection of the completed unit behind the rectangle may be seen differently as a hole and a figure respectively.

Surface completion, as described here, is really not a new notion, although disentangling it from boundary determination may prove useful. Rubin's (1915) classic observations about figure-ground organization implicate such a completion process. Rubin argued that when an area is projectively surrounded by another surface, the surrounded area—seen as figure—"owns" the bounding contour, while the surrounding surface continues behind the figure. This rule of enclosing and enclosed area, like the other rules enumerated by Rubin, are fairly weak influences that seem most applicable to pictorial displays. More commonly in three-dimensional perception, depth information such as binocular disparity or accretion–deletion of texture during observer motion specifies the relative depth of surfaces, and, accordingly, which surface is bounded by the contour. A result of this boundary assignment is the continuation of the further surface behind the nearer.

Related phenomena have been reported in the modal case, where completed surfaces appear in front of others, rather than behind. Walls (1954) reviewed a number of cases of "filling in" phenomena. These include completion across the blindspot in each eye, the fovea under scotopic conditions, the central fovea when viewing blue fields (due to the absence of short-wavelength cones in that area), and scotomata phenomena connected to migraines or other disorders. Yarbus (1967; see also Krauskopf, 1963) performed a series of elegant experiments involving retinal stabilization of parts of the visual field. When the contours defining a blue circle on a red field, for example, were stabilized, they soon disappeared. At that point, the blue area was filled in perceptually with red. In perhaps the most striking demonstration of the active nature of this filling-in process, Yarbus showed that a subsequent change of the surround, e.g., from red



(a)

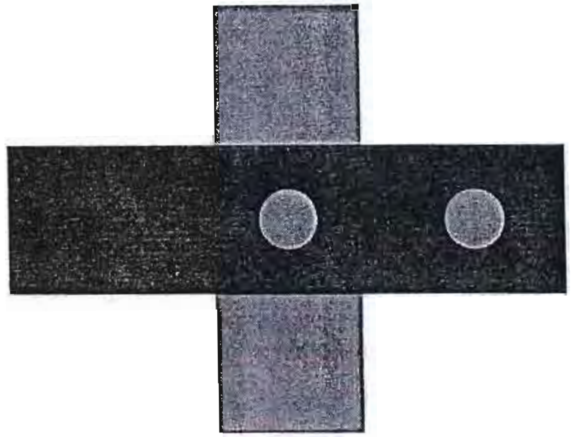


(b)

FIG. 49. Illustration of the surface completion process. (a) The central area is seen as a figure in front of the nearer rectangle. (b) A similar circle with surface color identical to the far rectangle is seen as a hole in the surface of the near rectangle. (c) Two identical circles appear differently (as a figure or a hole) depending on their placement relative to boundaries formed by the unit formation process.

to yellow, resulted in a persistence for several seconds of the red circle! It then changed to match the yellow surround.

What these phenomena have in common is the extending of perceived surface quality to unspecified regions. This seems to occur within but not across boundaries defining separate units (e.g., Yarbus, 1967). Grossberg & Mingolla (1985a) suggest a similar spreading of surface quality (the



(c)

FIG. 49—*Continued*

"feature contour" process) which operates between established boundaries.

It is too early to tell whether the "filling-in process" in these modal cases is identical to surface completion for partly occluded surfaces as illustrated in Fig. 49. In favor of such an identity is the obvious analogy with the process of boundary formation. Just as partly occluded object perception and illusory figure perception arise from a single mechanism, but differ in depth placement, so might the filling-in process be the same in its amodal and modal manifestations, again differing in terms of depth relations with other surfaces. Weighing against this idea might be the very considerations that led Michotte to label perception of occluded objects "amodal" in the first place. The partly hidden areas of perceived objects need not have any specific surface attributes. Figure 37 gave an example of unit formation where there is no clear assignment of specific qualities to the occluded regions. However, the fact that outcomes are sometimes indeterminate does not rule out the possibility that the filling-in process for surface quality often operates in the occlusion case.

In any event, there has not been much study of the rules governing surface completion in either the modal or amodal cases. For example, although complete enclosure seems to ensure the spread of a surface up to object boundaries, what happens when one surface partially, but not fully, encloses another (projectively)? Moreover, it is not known whether the surface completion process ever interacts with boundary formation to "extend" the figure, as might be occurring in Fig. 50. Working out the

FIG. 50. A possible interaction between the boundary formation and surface completion processes. The presence of the circle of similar surface quality may tend to extend the top rectangle behind the other rectangle.

specifics of surface completion, besides being important in its own right, may shed some light on issues involving boundary formation.

As this discussion of residual problems makes clear, much remains to be learned about the process of unit formation. The basic notions of a discontinuity theory, however, appear to unify and clarify many facts of object and contour perception. The approach has already led to a rich set of predictions and experimental studies, as well as the discovery of some novel object perception phenomena. The theory frames issues in ways that can guide further investigation. Perhaps most important in this regard is the promise for extending what is known about unit formation, mostly in static, two-dimensional cases, to a more general understanding of how observers perceive three-dimensional layouts and utilize information given by their own and objects' motion.

APPENDIX A

The Generic Occlusion

Suppose that an object with a smooth surface occludes another object with a smooth surface. Then, with probability one, there appears to be a corner, concave inwards, at the point of occlusion.

Proof: We consider points on the outlines of the objects, that is, points on the objects whose tangent planes pass through the eye.

A perceived "point" of occlusion is a line through the eye which is tangent to both objects; the tangent planes at the two points where this line touches the objects are denoted P_1 and P_2 in Fig. A1 (they are seen side-on since they pass through the eye). The perceived angle at which the outlines meet is the angle between these planes. It equals the angle between the projections of the outlines onto the retina, since the angle between the planes is the same at the retina as it is elsewhere.

It is possible for the planes P_1 and P_2 to coincide (for example, in the case of osculating spheres), but generically—that is, with probability one—they are different. (For, the planes coincide if their angles of elevation θ_1, θ_2 above the horizontal are equal, and the condition $\theta_1 = \theta_2$ determines a line in the two-dimensional θ_1–θ_2 space (Fig. A2). Hence, the probability that an arbitrary pair of angles θ_1, θ_2 falls on the line $\theta_1 = \theta_2$ is zero. Thus, with

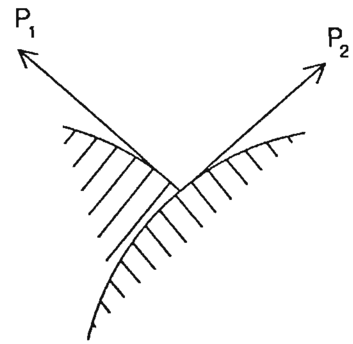


FIG. A1. Tangent planes to two objects at a point of occlusion (see text).

probability one, $\theta_1 \neq \theta_2$. The situation is an instance of the theorem of Guillemin and Pollack (1974, p. 30), where Y is the space of the retina, and X and Z are the projections on the retina of the outlines; the theorem of p. 40 implies that the situation occurs with probability one.)

Now, if $\theta_1 \neq \theta_2$, there is perceived an angle concave in to the figure, since, at the point of occlusion, opaque bodies occupy three of the four quarter-planes into which P_1 and P_2 divide the space.

This completes the proof.

Note: The proof may easily be extended to the (more realistic) case where the surfaces of the objects are only piecewise smooth. For the points at which the surfaces are not smooth form a finite set of curves, which project onto the retina also as a finite set of curves. Therefore, generically, a point of occlusion lies off all these curves, and so the situation is as before.

J. Franklin
School of Mathematics
The University of New South Wales

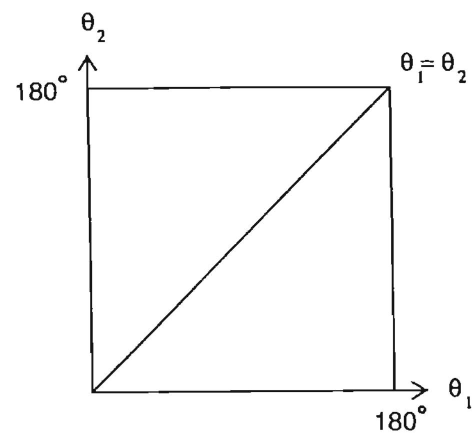


FIG. A2. Graph of possible coincident tangent planes of two objects in Θ_1–Θ_2 space, where Θ_1 and Θ_2 are the angles of planes 1 and 2 above the horizon (see text).

APPENDIX B

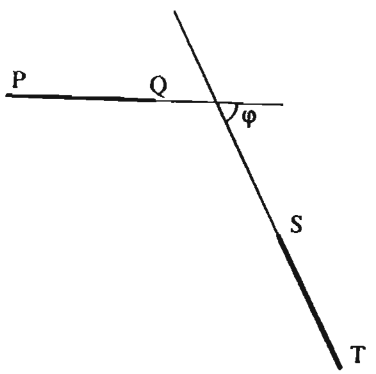
An Elementary Construction of a First-Order Continuous Curve Joining Two Relatable Edges

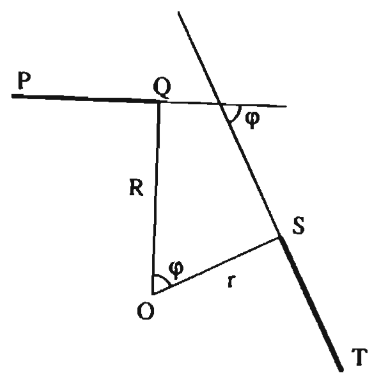
We begin with two relatable edges PQ and ST as indicated in Fig. B1 below. (Recall that relatability requires that the linear extensions of PQ and ST meet *in their extended domains* and that the angle φ indicated in Fig. B1 satisfies $0 \leq \varphi \leq \pi/2$.)

We wish to construct a first order continuous curve γ joining Q to S whose tangent at Q agrees with the line segment PQ and whose tangent at S agrees with the line segment ST.

In what follows we will consider only the case $\varphi > 0$. (If $\varphi = 0$, then the line segments are parallel and their common linear extension provides a natural first order continuous curve joining them.) We begin by constructing perpendiculars to PQ at Q and to ST at S. If $\varphi > 0$, these perpendiculars meet at a point O which is at a distance R from Q and r from S (see Fig. B2). For definiteness we will assume that $r \leq R$.

Note (by similar triangles) that the angle of φ of Fig. B1 is equal to the angle QOS of Fig. B2. Note also that $0 \leq R \cos \varphi < r$. In particular, if $\varphi > 0$, then "relatability," together with the above construction, leads to the condition $0 \leq R \cos \varphi < r$. As an aside we note that *the converse is also true*. In other words, given two nonintersecting and nonparallel line segments, a positive angle φ can be defined more or less as in Fig. B1, and the construction of





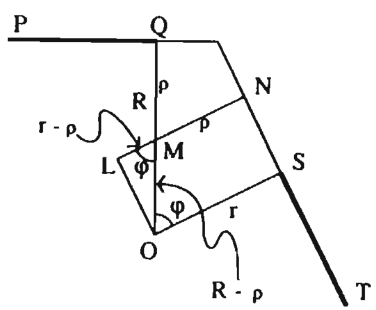


FIG. B1. Two relatable edges and their extensions (see text).
FIG. B2. The construction defining relatability (see text).
FIG. B3. Elaborated construction from B2 for deriving a curve having radius of curvature ρ or ∞ at all points (see text).

Fig. B2 leading to the quantities r and R, with $r \leq R$, can be accomplished. In this case the condition $0 \leq R \cos \varphi < r$ implies (1) that $0 < \varphi \leq \pi/2$ and (2) that the extended lines meet in their extended domains, i.e., the two lines are relatable. (Alternatively, this equivalence could be restated using equalities throughout, i.e., $0 \leq R \cos \varphi \leq r$, provided we agree to include the degenerate or "borderline" case $R \cos \varphi = r$ in which the extended lines meet at an end point of their extended domains and also agree that, if $\varphi = 0$, then $R = r = \infty$. It is formally convenient to do this in order to include the case $\varphi = 0$ as is done in the body of the text.)

Now consider the point M on the line segment OQ that is equidistant from Q and from the (extended) line segment ST (see Fig. B3). If $R = r$ then M and O coincide. Otherwise M is intermediate between O and Q.

Say the point M is a distance ρ from Q and from the point N nearest to it on the (extended) line segment ST. Necessarily MN is perpendicular to ST (extended).

Construct L on MN (extended) so that OL is perpendicular to LM and consider the right triangle OLM. The side LM has length $r - \rho$. The hypotenuse OM has length $R - \rho$. The angle in between is φ. Thus $\cos \varphi = (r - \rho)/(R - \rho)$. Solving for ρ leads to the equation

$$\rho = (r - R \cos \varphi)/(1 - \cos \varphi).$$

The curve γ *is constructed as follows:* Take as the first part of γ the circle of radius ρ centered at M starting at the point Q and ending at the point N. The second part of γ is just the straight line segment from N to S. Clearly γ has a continuous tangent and has the desired tangents at the points Q and S. Furthermore the radius of curvature is either r or infinite at each point of γ.

It is perhaps worth noting that the quantity ρ varies directly as the quantity $r - R \cos \varphi$, which enters into the characterization of relatability, and varies inversely as the quantity $1 - \cos \varphi$, which will be small when φ is close to 0. A small ρ indicates a sharp "bend" in the curve γ.

The curve γ appears to have the following rather interesting property: Any other first order continuous curve joining the line segments PQ and ST must have some points where the radius of curvatures is less than or equal to ρ. In other words, any other such curve will have a bend in it that is at least as sharp as the bend anywhere on γ. In this sense γ appears to be a "best possible" curve joining the two line segments, with r as the largest possible minimum curvature. This notion is slightly different from the notion of "minimum total curvature" used by Ullman (1976).

J. Edward Skeath
Department of Mathematics
Swarthmore College

REFERENCES

Andersen, G. J., & Cortese, J. M. (1989). 2-D contour perception resulting from kinetic occlusion. *Perception & Psychophysics*, 46(1), 49–55.

Attneave, F., & Arnoult, M. D. (1956). The quantitative study of shape and pattern perception. *Psychological Bulletin*, 53, 452–471.

Bertenthal, B. I., Campos, J. J., & Haith, M. M. (1980). Development of visual organization: The perception of subjective contours. *Child Development*, 51, 1072–1080.

Bradley, D. R. (1987). Cognitive contours and perceptual organization. In S. Petry & G. Meyer (Eds.), *The perception of illusory contours*. New York: Springer-Verlag.

Bradley, D. R., & Petry, H. M. (1977). Organizational determinants of subjective contour: The subjective Necker cube. *American Journal of Psychology*, 90, 253–262.

Brady, M., & Grimson, W. E. L. (1981). The perception of subjective surfaces. MIT, AI Memo No. 666.

Braunstein, M. (1976). *Depth perception through motion.* New York: Academic Press.

Brigner, W. C., & Gallagher, M. B. (1974). Subjective contour: Apparent depth or simultaneous brightness contrast? *Perceptual and Motor Skills,* 38, 1047–1053.

Brussell, E. M., Stober, S. R., & Brodinger, D. M. (1977). Sensory information and subjective contours. *American Journal of Psychology,* 90, 145–156.

Buffart, H., Leeuwenberg, E., & Restle, F. (1981). Coding theory of visual pattern completion. *Journal of Experimental Psychology: Human Perception and Performance,* 7(2), 241–275.

Coren, S. (1972). Subjective contours and apparent depth. *Psychological Review,* 79, 359–367.

Day, R. H. (1987). Cues for edge and the origin of illusory contours: An alternative approach. In S. Petry & G. E. Meyer (Eds.), *The perception of illusory contours.* New York: Springer-Verlag.

Day, R. H., & Jory, M. K. (1978). Subjective contours, visual acuity, and line contrast. In J. C. Armington, J. E. Krauskopf, & B. R. Wooten (Eds.), *Visual psychophysics: Its physiological basis.* New York: Academic Press.

Day, R., & Kasperczyk, R. (1983). Illusory contours in line patterns with apparent depth due to either perspective or overlay. *Perception,* 12, 485–490.

Dumais, S., & Bradley, D. (1976). The effect of illumination level and retinal size on the apparent strength of subjective contours. *Perception & Psychophysics,* 19, 339–345.

de Weert, C. M. M. (1987). Subjective contour strength and perceptual superposition: Transparency as a special case. In S. Petry & G. E. Meyer (Eds.), *The perception of illusory contours,* pp. 165–170. New York: Springer-Verlag.

Frisby, J. P., & Clatworthy, J. L. (1975). Illusory contours: Curious cases of simultaneous contrast? *Perception,* 4, 349–357.

Gans, D. (1969). Transformations and geometries. New York: Appleton–Century–Crofts.

Gerbino, W., & Kanizsa, G. (1987). Can we see constructs? In S. Petry & G. Meyer (Eds.), *The perception of illusory contours.* New York: Springer-Verlag.

Gibson, J. J. (1966). *The senses considered as perceptual systems.* Boston: Houghton Mifflin.

Gibson, J. J. (1979). *The ecological approach to visual perception.* Boston: Houghton Mifflin.

Gibson, J. J., Kaplan, G. A., Reynolds, H. N., & Wheeler, K. (1969). The change from visible to invisible: A study of optical transitions. *Perceptions & Psychophysics,* 5, 113–116.

Gilchrist, A., Delman, S., & Jacobsen, A. (1983). The classification and integration of edges as critical to the perception of reflectance and illumination. *Perception & Psychophysics,* 33(5), 425–436.

Gillam, B. (1972). Perceived common rotary motion of ambiguous stimuli as a criterion of perceptual grouping. *Perception & Psychophysics,* 11, 99–101.

Gillam, B. (1981). Separation relative to length determines the organization of two lines into a unit. *Journal of Experimental Psychology: Human Perception and Performance.* 7(4), 884–889.

Gillam, B. (1987). Perceptual grouping and subjective contours. In S. Petry & G. E. Meyer (Eds.), *The perception of illusory contours,* pp. 268–273. New York: Springer–Verlag.

Gillam, B., & Grant, T. (1984). Aggregation and unit formation in the perception of moving collinear lines. *Perception,* 13, 659–664.

Gillam, B., & McGrath, D. (1979). Orientation relative to the retina determines perceptual organization. *Perception & Psychophysics,* 26, 177–181.

Gregory, R. L. (1972). Cognitive contours. *Nature (London)* 238, 51–52.

Gregory, R. L. (1977). Vision with isoluminant colour contrast: I. A projection technique and observations. *Perception,* 6, 113–119.

Gregory, R. L. (1987). Illusory contours and occluding surfaces. In S. Petry & G. E. Meyer (Eds.), *The perception of illusory contours.* New York: Springer-Verlag, 81–89.

Gregory, R. L., & Harris, J. (1974). Illusory contours and stereo depth. *Perception & Psychophysics,* 15, 411–416.

Grossberg, S. (1987). Cortical dynamics of three-dimensional form, color, and brightness perception: I. Monocular theory. II. Binocular theory. *Perception & Psychophysics,* 41(2), 87–158.

Grossberg, S., & Mingolla, E. (1985a). Neural dynamics of form perception: Boundary completion, illusory figures, and neon color spreading. *Psychological Review,* 92(2), 173–211.

Grossberg, S., & Mingolla, E. (1985b). Neural dynamics of perceptual grouping: Textures, boundaries, and emergent segmentations. *Perception & Psychophysics,* 38(2), 141–171.

Grossberg, S., & Mingolla, E. (1987a). The role of illusory contours in visual segmentation. In S. Petry & G. E. Meyer (Eds.), *The perception of illusory contours.* New York: Springer-Verlag.

Grossberg, S., & Mingolla, E. (1987b). Neural dynamics of form perception: Boundary webs, illuminants, and shape-from shading. *Computer Vision, Graphics, and Image Processing,* 37, 116–165.

Guillemin, V., & Pollack, A. (1974). *Differential topology.* Englewood Cliffs, NJ: Prentice-Hall.

Hochberg, J. (1971). Perception: II. Space and movement. In J. W. Kling & L. A. Riggs (Eds.), *Handbook of Experimental Psychology,* 475–550.

Hochberg, J. (1978). *Perception.* Englewood Cliffs, NJ: Prentice-Hall.

Hochberg, J., & Brooks, V. (1960). The psychophysics of form: Reversible-perspective drawings of spatial objects. *American Journal of Psychology,* 73, 337–354.

Hoffman, D. D., & Richards, W. A. (1984). Parts of recognition. *Cognition,* 18, 65–96.

Johansson, G. (1970). On theories for visual space perception. *Scandinavian Journal of Psychology,* 11, 67–74.

Johansson, G. (1975). Visual motion perception. *Scientific American,* 232, 76–89.

Jory, M., & Day, R. (1979). The relationship between brightness contrast and illusory contours. *Perception,* 8, 3–9.

Kanizsa, G. (1955). Margini quasi-percettivi in campi con stimolazione omogenea. *Rivista di Psicologia,* 49, 31–49.

Kanizsa, G. (1979). *Organization in vision.* New York: Praeger.

Kaplan, G. (1969). Kinetic disruption of optical texture: The perception of depth at an edge. *Perception & Psychophysics,* 6, 193–198.

Kaufmann-Hayoz, R., Kaufmann, F., & Walther, D. (1988). Perception of kinetic subjective contours at 5 and 8 months. *Infant Behavior & Development,* 11, 168. [Abstract]

Kellman, P. J., & Cohen, M. H. (1984). Kinetic subjective contours. *Perception & Psychophysics,* 35(3), 237–244.

Kellman, P. J., Gleitman, H., & Spelke, E. S. (1987). Object and observer motion in the perception of objects by infants. *Journal of Experimental Psychology: Human Perception and Performance,* 13(4), 586–593.

Kellman, P. J., & Loukides, M. G. (1986). Approaching the edge: An object perception view of static and kinetic subjective contours. *Perception & Psychophysics,* 39(3), 216–217. [Abstract]

Kellman, P. J., & Loukides, M. G. (1987). An object perception approach to static and kinetic subjective contours. In S. Petry & G. E. Meyer (Eds.), *The Perception of Illusory Contours.* New York: Springer-Verlag.

Kellman, P. J., Power, L., & Shipley, K. S. (1989, June). *Visual interpolation in object perception: Evidence from a kinetic occlusion paradigm*. Paper presented at the Fifth International Conference on Event Perception and Action, Oxford, Ohio.

Kellman, P. J., & Short, K. R. (1985, June). Infant object perception from motion information: The problem of rotation. Paper presented at the Third International Conference on Event Perception and Action, Uppsala, Sweden.

Kellman, P. J., & Shipley, T. F. (1987, November). *Interpolation processes in visual object perception: Evidence for a discontinuity theory*. Paper presented at the 28th annual meeting of The Psychonomic Society, Seattle.

Kellman, P. J., & Spelke, E. S. (1983). Perception of partly occluded objects in infancy. *Cognitive Psychology*, 15, 483–524.

Kellman, P. J., Spelke, E. S., & Short, K. (1986). Infant perception of object unity from translatory motion and motion in depth. *Child Development*, 57(1), 72–86.

Kennedy, J. M. (1976). Sun figure: An illusory diffuse contour resulting from an arrangement of dots. *Perception*, 5, 479–481.

Kennedy, J. M. (1978a). Illusory contours not due to completion. *Perception*, 7, 187–189.

Kennedy, J. M. (1978b). Illusory contours and the ends of lines. *Perception*, 7, 605–607.

Kennedy, J. M. (1988). Line endings and subjective contours. *Spatial Vision*, 3(3), 151–158.

Koffka, K. (1935). *Principles of Gestalt psychology*. New York: Harcourt Brace.

Krauskopf, J. (1963). Effect of retinal image stabilization on the appearance of heterochromatic targets. *Journal of the Optical Society of America*, 53, 741–744.

Leclerc, Y. G., & Zucker, S. W. (1987). Local structure of image discontinuities in one dimension. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 9, 341–355.

Livingstone, M., & Hubel, D. (1987). Psychophysical evidence for separate channels for the perception of form, color, movement and depth. *Journal of Neuroscience*, 7(11), 3416–3468.

Marr, D. (1982). *Vision*. San Francisco: Freeman.

Marr, D., & Hildreth, E. (1980). Theory of edge detection. *Proceedings of the Royal Society of London, B*, 207, 187–217.

Metelli, F. (1974). Achromatic color conditions in the perception of transparency. In R. B. MacLeod & H. L. Pick, Jr. (Eds.), *Perception: Essays in honor of James J. Gibson*. Ithaca, NY: Cornell Univ. Press.

Michotte, A., Thines, G., & Crabbe, G. (1964). Les complements amodaux des structures perceptives. *Studia Psychologica*. Louvain: Publications Universitaires de Louvain.

Minguzzi, G. F. (1987). Anomalous figures and the tendency to continuation. In S. Petry & G. Meyer (Eds.) *The Perception of Illusory Contours*. New York: Springer-Verlag.

Parks, T. E. (1984). Illusory figures: A (mostly) a theoretical review. *Psychological Bulletin*, 95, 282–300.

Parks, T. E. (1986). Illusory figures, illusory objects, and real objects. *Psychological Review*, 93(2), 207–215.

Petry, S., Harbeck, A., Conway, J., and Levey, J. (1983). Stimulus determinants of brightness and distinctness of subjective contours. *Perception & Psychophysics*, 34(2), 169–174.

Prazdny, K. (1983). Illusory contours are not caused by simultaneous brightness contrast. *Perception & Psychophysics*, 34, 403–404.

Prazdny, K. (1985). On the nature of inducing forms generating perception of illusory contours. *Perception & Psychophysics*, 37, 237–242.

Prazdny, K. (1986). Illusory contours from inducers defined solely by spatiotemporal correlation. *Perception & Psychophysics*, 39(3), 175–178.

Robinson, J. O. (1972). *The psychology of visual illusions*. London: Hutchinson.

Rock, I. (1983). *The logic of perception*. Cambridge, MA: MIT Press.

Rock, I. (1987). A problem solving approach to illusory contours. In S. Petry & G. E. Meyer (Eds.), *The perception of illusory contours*. New York: Springer-Verlag.

Rock, I., & Anson, R. (1979). Illusory contours as the solution to a problem. *Perception*, 8, 665–687.

Rubin, E. (1915). *Synsoplevede figurer*. Copenhagen: Gyldendals.

Sambin, M. (1974). Angular margins without gradients. *Italian Journal of Psychology*, 1, 355–361.

Sambin, M. (1987). A dynamic model of anomalous figures. In S. Petry & G. E. Meyer (Eds.), *The Perception of Illusory Contours*. New York: Springer-Verlag.

Schumann, F. (1904). Einige Beobachtungen Uber die Zusammenfassung von Gesichtseindruckern zu Binheiten. *Psychologische Studien*, 1, 1–32.

Shepard, R. (1984). Ecological constraints on internal representation: Resonant kinematics of perceiving, imaging, thinking and dreaming. *Psychological Review*, 91, 441–447.

Shipley, T. F. (1988). *Unit formation in subjective contour perception: Evidence for a discontinuity theory*. Unpublished doctoral dissertation, University of Pennsylvania.

Shipley, T. F., & Kellman, P. J. (1988, November). *Discontinuity theory and the perception of illusory figures*. Paper presented at the 29th Annual Meeting of the Psychonomic Society, Chicago.

Shipley, T. F. & Kellman, P. J. (1990). The role of discontinuities in the perception of subjective figures. *Perception & Psychophysics*, 48(3), 259–270.

Shipley, T. F., & Kellman, P. J. (in press). Perception of partly occluded objects and illusory figures: Evidence for an identity hypothesis. *Journal of Experimental Psychology: Human Perception & Performance*.

Shipley, T. F., & Kellman, P. J. (manuscript in preparation). The effect of symmetry on the formation of illusory figures.

Todd, J. (1982). Visual information about rigid and nonrigid motion: A geometric analysis. *Journal of Experimental Psychology: Human Perception & Performance*, 8, 238–252.

Ullman, S. (1976). Filling in the gaps: The shape of the subjective contours and a model for their generation. *Biological Cybernetics*, 25, 1–6.

Ullman, S. (1979). *The interpretation of visual motion*. Cambridge, MA: MIT Press.

Ullman, S. (1984). Visual routines. *Cognition*, 18, 97–159.

van Tuijl, H., & Leeuwenberg, E. (1982). Peripheral and central determinants of subjective contour strength. In H. G. Geissler, P. Petzoldt, H. F. J. M. Buffart, & Y. M. Zabrodin (Eds.), *Psychological judgment and the process of perception*. Amsterdam: North-Holland.

von Hornbostel, E. M. (1927). Die einheit der sinne. *Melos*, 4, 290–297.

Wallach, H., & Slaughter, V. (1988). The role of memory in perceiving subjective contours. *Perception & Psychophysics*, 43, 101–106.

Wallach, H., & O'Connell, D. N. (1953). The kinetic depth effect. *Journal of Experimental Psychology*, 45, 205–217.

Walls, G. (1954). The filling-in process. *American Journal of Optometry and Archives of American Academy of Optometry*, 31(7), 1954.

Ware, C. (1980). Coloured illusory triangles due to assimilation. *Perception*, 9, 103–107.

Wertheimer, M. (1912). Experimentelle studien uber das sehen von bewegung. *Zeitschrift fur Psychologie*, 61, 161–265.

Wertheimer, M. (1923). Untersuchungen zur Lehre von der Gestalt. *Psychologische Forschung*, 4, 301–350.

Yarbus, A. L. (1967). *Eye movements and vision*. New York: Plenum.

Yonas, A., & Granrud, C. (1984). The development of sensitivity to kinetic, binocular and pictorial depth information in human infants. In D. Ingle, D. Lee, & M. Jeannerod (Eds.), *Brain mechanisms and spatial vision*. Amsterdam: Nijhoff.

(Accepted May 9, 1990)