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Developing New Approaches to Mathematics Learning in the Classroom

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Most humans, both young and old, are capable of remarkable feats of learning in their everyday lives, and yet, all too often, the news from classrooms in the United States is about perennial difficulty and persistent failure for large numbers of students in achieving the learning goals set out in local, state, and national standards. Although the causes and potential cures are many and varied, in this chapter we consider an approach that addresses dimensions of learning that have been studied for decades in the learning sciences but have received little to no attention in K–12 classrooms. Specifically, we examine *perceptual learning* as a form of learning that contributes to the insight and fluency that characterize expertise across many settings and domains. We introduce what perceptual learning (PL) is, findings about PL that have emerged from several different lines of research, and how PL might be brought into K–12 classrooms as a significant complement to other forms of instruction. Two empirical studies of PL interventions using specially designed learning software, known as Perceptual Learning Modules (PLMs), illustrate some of its key characteristics and effects on students' learning in mathematics.

What Is Perceptual Learning?

The classic definition of perceptual learning, offered by Eleanor Gibson (1969), is "an increase in the ability to extract information from the environment, as a result of experience and practice with stimulation coming from it" (p. 3). With practice, in virtually all domains of human experience, people become significantly better and faster at extracting relevant information, ignoring irrelevant information, making fine discriminations, and perceiving higher-order structure and relationships. It is particularly noteworthy that PL emphasizes the pick-up of information that is demonstrably present in the external environment (though typically unnoticed or inefficiently processed by novices). The changes that result from such learning are changes in the ability to recognize or distinguish significant features, structures, or relationships. (For a more detailed discussion of PL in relation to other taxonomies of learning see Gibson & Gibson, 1955, and Kellman & Garrigan, 2008. See Kellman, 2002, and Goldstone, 1998, for general reviews of contemporary research on PL.)

PL involves the optimization of attention, so that the learner becomes increasingly selective in what information is attended to and what is disregarded. It is also characterized by increasing specificity of discrimination, such that experience allows the learner to make fine distinctions among features or structures that initially appeared to be the same. Although the term "perceptual" may evoke a sense that the information being extracted involves only low-level sensory inputs, in concert both with contemporary theories of perception (J. Gibson, 1979; Marr, 1982) and with classic treatments of PL (E. Gibson, 1969) we see perception and perceptual learning as being crucially involved with the extraction of higher-level invariants and abstract structures. Indeed, an aim of this chapter is to extend applications of PL to the abstract symbolic domain of mathematics.

Perceptual Learning and Expertise

Studies of expertise across a variety of domains have provided a particularly rich set of phenomena illustrating the power of PL. Studies in diverse areas, including aviation (Kellman & Kaiser, 1994), radiology (Kundel & Nodine, 1975), and chess (Chase & Simon, 1973; DeGroot, 1965), have documented the development of the ability to detect and discriminate relevant features, patterns, and relationships, typically after extensive practice involving many examples in the course of carrying out a meaningful task. Experts differ from novices not just in their declarative knowledge or their procedural competence; one of the most distinctive features of their expertise is that they seem to "just see" (or hear, taste, etc.) differences, patterns, structures, and relationships in the environmental input that either are unavailable to novices or require effortful and error-prone analysis.

Whereas the novice typically attends to superficial or irrelevant information as much as relevant information, the expert's attention is selective. Experts tend to process both very fine-grained levels of detail when relevant, as well as abstract higher-order "chunks" and relationships (Chi, Feltovich, & Glaser, 1981). Experts are also more fluent and may show automaticity (Schneider & Shiffrin, 1977), processing remarkably large amounts of task-relevant information quickly and effortlessly in a way that is relatively insensitive to cognitive load.

Perceptual Learning in Everyday Contexts

Perceptual learning does not arise only in esoteric domains involving advanced training and education; it underlies many capacities that we take for granted in our everyday lives. Our ability to learn to recognize the faces of students in a class or to immediately identify the voices of friends and family on the phone are instances of PL.

It is likely that PL underlies our acquisition of many everyday concepts, as well. Consider, for instance, the case of a young child learning the extension of the word "dog." The child's task is to learn to pick out all and only the dogs in the world, despite (1) the very great variety among different types of dogs (for instance, a Saint Bernard vs. a hairless Chinese Crested); and (2) the strong resemblance between some dogs and animals of other species (e.g., a German Shepherd and a wolf, or a furry Pomeranian and a fluffy Persian cat). Restated as a more general principle of PL, the child must learn to extract the invariances that underlie the different instances of the target category or concepts, and must also learn to discriminate the key structures from negative cases that may appear similar, but which do not belong to the category. Very young children spontaneously engage in this kind of learning

and become remarkably proficient without much in the way of explicit or systematic instruction.

What Promotes Perceptual Learning?

Interestingly, even an accomplished adult learner may or may not be able to articulate the grounds on which he or she recognizes and discriminates key structures and relationships. Experimental studies of PL using unfamiliar laboratory stimuli indicate that learners may approach perfect performance in being able to detect or discriminate target structures without being able to explain how they are doing so. Moreover, didactic instruction is generally not an effective way to promote PL. Instead, what seems to promote PL most effectively is processing many examples of instances that possess the features, structures, or relationships of interest and seeing them across a widely varying background of contexts and contrasting cases. This kind of experience may accrue naturally as an individual spends much time immersed in the practice of a hobby or profession, or it can be deliberately structured in a training or educational program.

Perceptual Learning in Middle-School Mathematics Instruction

Accelerating Expertise: Perceptual Learning Modules

In the intervention studies that follow, we use custom-designed learning software known as Perceptual Learning Modules (PLMs) as a way to operationalize a learning environment based on PL principles. PLMs rest on the assumption that human perceptual learning abilities advance not from the passage of time, the acquisition of declarative verbal knowledge, or the rehearsal of fixed procedures, but from making discriminations and classifications relevant to some task. The critical learning activity in the designed PLMs involves asking the learner to recognize or discriminate a target relational structure or to map related structures across different representations (e.g., graphs and equations) or across transformations (e.g., algebraic transformations performed on an equation) (Kellman et al., 2008). Several principles, derived from the research literature on PL, guide the way we structure materials and activities to make learning trials effective.

First, the PLM software is designed to engage the learner in *large numbers of brief classification episodes*—not just a small number of examples. This approach departs from common practice in math classrooms in two notable ways. First, learners see many more instances of the target structures and relationships and in more contexts than would normally be presented in a typical scenario (where a teacher works one or two problems with the whole class and students then go on to work problems that are similar to the model).

Second, when PL is the instructional goal, a much greater percentage of students' time and effort is devoted to *problem recognition and classification*, rather than completing calculations and procedures to solve problems. Learning episodes go quickly; a student might complete a dozen or more trials in the time it would take to work a problem by completing all of the calculations to arrive at a solution. Because specific instances seldom or never repeat in PLMs, structural invariance is learned, and learning generalizes to new instances.

It is also critical that the learning trials present multiple instances that incorporate *several kinds of systematic variation* across classification episodes. First, invariant relations characterizing key structure must appear in a variety of contexts so that the learner will come to recognize these structures independent of local contexts or surface variations. Second, irrelevant aspects of problems need to vary, so as not to be mistakenly correlated with the target structure. Contrasting classifications from which the learner must differentiate the target structure(s) should also appear. This is especially important when the learning involves differentiating similar structures, since learning to discriminate among a set of items that at first seem all to look alike is a frustrating learning problem commonly faced by novices. What is more, this learning problem is commonly underestimated by experts who have already automatized the discriminations.

What Perceptual Learning Is Not

It is important to consider approaches to learning that contrast with PL. Because of the large number of learning trials and the emphasis on having the learner interact with them, PL might be confused with some other well-known approaches. Although learners using the PLMs see many individual trials, the goal of the learning is not to memorize individual instances or a fixed set of items. The learning sets are typically very large, and the goal is to enable the learner to recognize or map the targeted structures in new contexts and novel instances.

PLM trials are also not intended to promote stimulus–response associations, as would be the goal in behaviorist approaches. Although having the learner actively engage with the items is often useful for maintaining attention and motivation, the specific behavioral responses employed are not the goal of the intervention. Instead, PL is aimed at the discovery and efficient extraction of important structures in novel, varying inputs. The fundamental question the learner is addressing in PL interventions is "What do I *see* in this problem?", not "What should I *do*?" This ability to recognize and discriminate between relevant structures and relationships may be incorporated into any number of behavioral repertoires, including complex problem solving. Indeed, we argue that the gains in being able to quickly and accurately select the most useful information for a meaningful task can reduce processing demands in a way that enables the learner to build up more sophisticated and flexible problem-solving strategies, as opposed to stereotyped behavioral routines.

The two studies that follow test the application of PL interventions, in the middle grades, to mathematics learning of fractions and measurement. These mathematical topics were selected for two reasons. First, they are areas in mathematics that many students fail to master despite their prominent representation in many curricula. Second, we believe that successful learning in these areas involves structure extraction, pattern recognition, and the achievement of fluency—thus making them good test cases for the kinds of learning for which PL interventions may provide a useful complement to other forms of instruction.

Study 1: Finding Structure in Fraction Problems

Many students struggle with word problems, and their difficulties often seem to increase when the problems involve fractions. Of course, successfully solving written problems involving rational numbers is a complex process requiring coordination of different kinds of knowledge and strategies, and there can be many sources of learning difficulties. This study investigates one aspect: whether instruction that emphasizes extracting underlying problem structures can help students learn to reliably discriminate between contrasting problem types and thus improve their ability to solve such problems.

Consider the following problems:

1 A class went on a 72-mile road trip. They stopped for lunch after driving three-quarters of the way. How many miles did they drive before stopping for lunch?
2 A class went on a road trip. After driving 72 miles, the driver announced that they had driven three-quarters of the way. How many miles are in their entire trip?

Both problems include the same quantities, involving a whole number (72 miles) and a fraction (3/4), which appear in the same order. Although the problems look alike, they have contrasting underlying structures. The first, which could be restated as "How many miles is three-quarters of 72 miles?," is what we term a "find the part" problem. The total quantity is known, and the question involves finding a fractional part of that total. The second, which could be stated in simplified form as "72 miles is three-quarters of how many miles?," is a "find the whole" problem, in which the amount of the fractional quantity is known and the task is to use that to figure out the total quantity. Many students fail to see the structural distinction between these two problems. They may see them as identical and attempt to solve them in the same way, or they may simply pull out the numbers and attempt to perform an operation on them, seemingly at random or based on the convenience of the calculation (Bell, Fischbein, & Greer, 1984). Our overarching hypothesis in this study is that actively recognizing the underlying problem structure and mapping that structure across word problems, graphic representations, and numerical representations (i.e., number sentences used to calculate a solution) are critical components in problem solving; furthermore, learning interventions that focus on structural relations and mapping will transfer to significant improvements in open-ended problem solving.

To examine whether PL approaches can help students improve at this kind of structural analysis and mapping, we developed a series of classroom lessons and two versions of a computer-based PL training module. Different versions of the PLM software were designed to address an additional experimental question related to the structure and sequence of how the content was introduced. It is common in many K–12 classrooms and curricula to break material down into component pieces and to build from simpler constituents to more complicated ones. Following this approach, we created one instructional sequence that first introduced unit fractions (fractions with a numerator of 1) and then introduced nonunit fractions. In a contrasting condition, students were introduced to both unit and nonunit fractions in classroom lessons and then used a version of the software that mixed unit and nonunit trials throughout. This approach to organizing instruction comes from the research literature on memory, motor learning, and training in industrial settings, where the issue of blocked versus randomized learning trials has been the subject of considerable study. In a review of a number of training studies, Schmidt and Bjork (1992) argue that mixing item types ultimately produces more durable learning as well as better transfer of learning, although it might slow the course of learning during training.

One group of students participated only in the classroom lessons, without using the PLM, while two other groups participated in both classroom lessons and individual PLM training. This design allowed us to ask whether teacher-led instruction that deliberately developed fraction concepts and problem-solving strategies from a structural point of view is effective at all in promoting students' learning and problem solving, and, if so, whether it is sufficient in itself or whether additional computer-based PL training could lead to further learning gains. We predicted that all groups participating in the study would improve, but that the groups in the PLM conditions would show larger or more durable learning gains. Motivating these predictions was the hypothesis that the PLM training would further consolidate students' ability to extract and map the target structures and would allow the development of fluency.

Method

PARTICIPANTS

Participants were 76 seventh-grade students (44 female, 32 male) enrolled in an urban public school serving a predominantly minority low-income population.

DESIGN AND PROCEDURE

All students completed a custom-designed pretest and were then randomly assigned to one of three conditions with the constraint that the means and ranges of pretest scores were approximately equal for each group. Students in all three conditions participated in a series of 16 classroom lessons during their regular math classes designed and led by Zipora Roth, an experienced middle-school math teacher and curriculum specialist. For students in the No-PLM Control group, there was no further learning intervention after the classroom lessons. Students in the Unit First PLM Condition completed the first sequence of seven classroom lessons, which involved unit fractions, then worked for several individual sessions with a version of the PLM software that contained only unit fraction problems. They then completed the last sequence of nine classroom lessons, which introduced nonunit fractions, before returning to individual sessions with PLM software that included both unit and nonunit problems. Students in the Mixed PLM Condition completed the entire sequence of 16 classroom lessons and then began training in individual sessions using a version of the PLM software that included both unit and nonunit fractions mixed from the beginning.

Students in the two PLM conditions worked individually for sessions of about 30–40 minutes approximately once per day until they either retired all categories by meeting predetermined speed and accuracy criteria or exhausted the time allowed for the study. Because the software adapts to each learner by allowing the student to retire problem categories as they are mastered, the number of sessions completed was highly variable, ranging from a minimum of two sessions (for students who demonstrated mastery quickly) to as many as 15 sessions. At the end of each group's learning intervention, students were given an immediate post-test. A delayed post-test was administered approximately nine weeks later, with no intervening study-related instruction.

MATERIALS

Classroom lessons Classroom instruction consisted of 16 lessons specially designed to provide a foundational introduction to fractions with a particular focus on structural relations underlying fraction concepts. Although the students in this school had completed curriculum units on fractions from current standards-based curricula, their performance on standardized assessments indicated that their math foundation was very weak and that a comprehensive review was in order. The lessons in this sequence introduced fractions involving both discrete and continuous quantities with activities emphasizing partitioning into units and iterating units to construct fractions to represent different quantities. Instruction also introduced four representations—word problems, simplified questions, fraction strips, and number sentences—that were also used in the PLM software.

PLM software The PLM software presented learners with many short trials wherein their task was to select which of several choices in one representational format matched a target presented in a different representational format. Any of the representational formats could appear in either target or choice positions. Choices were designed to include distractor items that represented common errors.

Designing trials so that the learner must map across representations challenges the learner to extract an abstract relational structure for which there are very few similarity cues. The choices, which were always of the same representational type, resembled each other much more than any one of them resembled the target. Trials were drawn from a large base of unique items, so that the user could not solve the task by memorizing answers to items that were repeated. Instead, on each trial, the student had to detect a common structure across stimuli with very different appearances (the target and its corresponding choice) and to discriminate among stimuli with similar appearances (the choices). Users received feedback on each trial as to whether they were correct or incorrect; if they were incorrect, the correct response was illustrated with a short interactive feedback sequence.

The PLM recorded each student's responses and reaction time on each trial and tracked their performance on various subcategories of problems against predetermined speed and accuracy criteria. Subcategories of items were based on bidirectional cross-mappings of each combination of representational types. When a student met the criteria for a given category of items (e.g., mapping word problems to corresponding fraction strips), that category was retired from the learning set.

PRETEST/POST-TEST FRACTION ASSESSMENT

Three equivalent forms of a 27-item pencil and paper test were created and administered in counterbalanced order as the pretest, immediate post-test, and delayed post-test. The assessment emphasized transfer items that did not directly resemble the kinds of tasks and problems used in either the classroom instruction or the PLM intervention. A few items were more similar to problems the students saw during training, but none was identical. The assessment included open-ended word problems requiring the students to calculate complete solutions, as well as multiple-choice items probing various aspects of their understanding of fractions, such as comparing and ordering fractions.

Results

The main results for this study are presented in Figure 16.1, which shows that students in all three groups demonstrated significant gains from pretest to immediate post-test and from pretest to delayed post-test. Students in the two PLM groups performed similarly to each other on the immediate post-test and outperformed the No PLM control group. By the time of the delayed post-test, the Mixed PLM group showed the best performance, fully maintaining learning gains over the nine-week delay. Mean scores on the delayed test for the Unit First PLM condition showed some decline from their immediate post-test scores. Students in the control group maintained their learning gains from immediate to delayed post-test, but did not reach levels as high as the Mixed PLM group.

These observations were tested statistically with a two-way repeated measures ANOVA performed on students' proportion of correct responses on the fraction assessment, with Test Phase (Pretest, Immediate Post-test, Delayed Post-test) as a within-subjects factor and Condition (Unit First PLM, Mixed PLM, and No PLM Control) as a between-subjects factor. There was a main effect of Test Phase [$F(2, 138) = 89.66$, $p < .001$], confirming that students across all groups showed significant learning gains. There was no reliable main effect of Condition, but there was a significant Condition by Test Phase interaction [$F(4, 138) = 5.396$, $p < .001$], indicating reliably different patterns of learning effects across conditions.

A series of planned comparisons was carried out to look at the condition differences in more detail. Both of the PLM groups showed significantly greater improvement

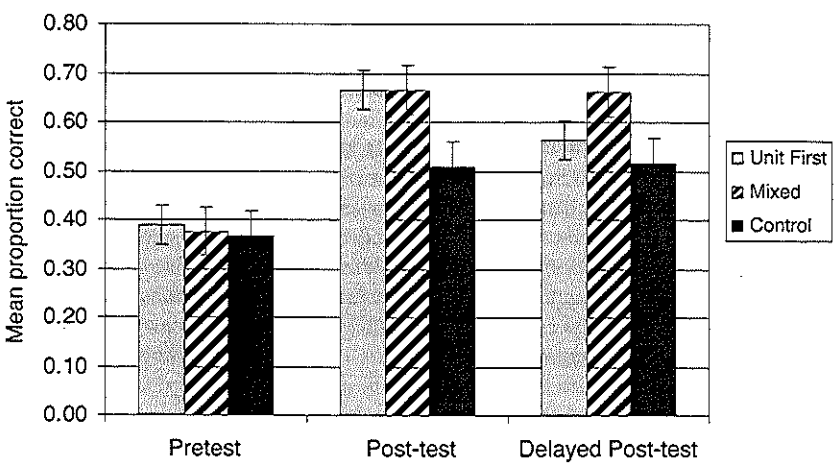

Figure 16.1 Mean Accuracy by Condition and Time of Test on the Fraction Assessment in Study 1. Error bars indicate ± one standard error of the mean. Adapted from Kellman, P. J., Massey, C., Roth, Z., Burke, T., Zucker, J., Saw, A., Aguero, K. E., & Wise, J. A. (2008). Perceptual learning and the technology of expertise: Studies in fraction learning and algebra. *Pragmatics & Cognition*, 16(2), 356-405.Copyright 2008 John Benjamins Publishing Company.

from Pretest to Immediate Post-test compared with the No PLM Control group [$t(51) = 2.60$, $p < .02$ for the Unit First PLM vs. Control, and $t(47) = 3.07$, $p < .01$, for the Mixed PLM vs. Control]. The two PLM groups did not differ from each other in improvement from Pretest to Immediate Post-test [$t(48) = .34$, n.s.], but the Mixed PLM group significantly outperformed both the Unit First PLM and the No PLM Control groups from Pretest to Delayed Post-test [$t(47) = 2.15$, $p < .04$, for the Mixed PLM vs. Unit First PLM groups and $t(43) = 2.86$, $p < .01$, for the Mixed PLM vs. No PLM Control groups]. The Unit First PLM group and the No PLM Control group did not differ significantly in learning gains from the pretest to the delayed post-test [$t(47) = -.528$, n.s.].

Discussion

The pattern of results obtained in this study generally supports our hypotheses that (a) instruction focused on structural patterns and relationships does lead to significant gains in students' ability to solve a variety of fraction problems in all conditions tested; and (b) supplementing classroom lessons with PLM training adds additional value to more typical classroom instruction methods. The classroom lessons included all of the same essential material, presented in a similar way—the PLM did not introduce any new content—yet students in the PLM conditions (particularly the Mixed PLM group) performed better than students in the No PLM Control group. Although the classroom lessons "covered" the material very systematically, most students had not learned it to the point that they demonstrated high levels of either accuracy or fluency when they first encountered the same ideas in the PLM. Both the classroom lessons and the software shared a focus on structural relationships and patterns as expressed in different representational formats. Critical differences between them were that students using the software saw a much larger and more varied set of examples. The PLM was also designed to help students extract target patterns and relationships on their own by interacting with them in a structured way, rather than having them explained by the teacher.

We submit that declarative and procedural components in instruction can be usefully supplemented by learning activities that focus specifically on practicing the extraction of important structural patterns and fluency. The number of PLM sessions completed by individual students was highly variable, indicating that students have very different needs for time and practice to consolidate this learning. PLM software holds significant promise as an efficient and effective way to customize instructional time to students' individual learning needs, and to track and certify to some standard the learning that is being accomplished.

The finding that students in the Mixed PLM group showed the strongest and most durable learning gains is also noteworthy. We suggest that mixing unit and nonunit fraction problems from the beginning of the PLM training may encourage learners to compare and contrast them and their relationship to a whole quantity from the beginning. That is, learners may be challenged to construct a more comprehensive and relational understanding at the outset rather than build what may be an overly simple approach to unit fractions that then has to be reworked and expanded when problems with nonunit fractions are introduced.

Study 2: Constructing Units of Linear Measurement

The second study we discuss extends our investigations using PL to another area known to be a problem in middle-school math classrooms in the United States: linear measurement. Many children come to measurement with mathematical conceptions tied to counting of discrete entities (Hartnett & Gelman, 1998). When they look at a ruler, they are looking for something to count and tend to focus on either the integers or the hash marks. This is related to the common observation in classrooms that children are extremely puzzled as to why one lines up the extent to be measured starting from the edge rather than from the number 1. Many children learn fixed procedures for measuring with rulers but continue to be confused about units of linear measurement and are unable to explain why the procedures are the way they are.

For example, when children are asked to measure with nonstandard rulers, they make characteristic errors. A compelling illustration comes from a released item from the 2003 National Assessment of Education Progress (National Center for Education Statistics, 2008) in which children were asked to measure a 2½-inch toothpick with a broken ruler that started at the 8-inch mark. Some students err by saying the toothpick is 10½ inches long. These students have probably learned a routine procedure for lining an item up with the end of the ruler and reading off the rightmost point. This kind of response indicates that their procedural knowledge is shallow, inflexible, and not supported by a mathematically correct understanding of units of linear measurement.

A more common error among both fourth and eighth graders is choosing 3½ inches as the correct answer. These students are most likely counting hash marks on the ruler, starting with the hash mark at the leftmost point of the toothpick as "1." Both of these incorrect responses suggest that students do not conceive of units of linear measurement as having *extent*. In other words, they do not recognize that an inch (or centimeter) on a ruler is the *entire length* between the hash marks that demarcate the unit, not just the point where the numbered hash mark is located.

A second learning challenge in elementary and middle-school mathematics is mastery of concepts related to fractions and operations with fractions. The National Mathematics Advisory Panel's (2008) recently released report singled out mastery of fractions as the most critical need in improving elementary mathematics education in the United States. Fractions and measurement are deeply connected—they involve the foundational mathematical concepts required for working with continuous quantities. It is through measurement, rather than discrete counting, that continuous quantities of all kinds (e.g., distance, area, volume, temperature, etc.) are numerically quantified (Schwartz, 1988); and accurate measurement frequently involves working with fractional parts of units (Lehrer, Jaslow, & Curtis, 2003). Indeed, measurement contexts can motivate meaningful understanding of fractions as numbers that represent quantities.

In the present study, we bring together linear measurement and fractions. The study involves a classroom intervention in which students participated in an introductory lesson and then worked individually with specially designed PLM software. The specific learning goals of this intervention were (1) understanding that any start point can be seen as zero on a scale; (2) distinguishing between position and distance on a ruler (e.g., starting at "3" as a position vs. moving three units as a distance); (3) measuring distances accurately and flexibly on a ruler, including fractional distances; (4) improving students' understanding of units of linear measurement, unit structure, and fractional partitions of units, as represented on rulers; and (5) strengthening students' intuitions about the relationship between ruler problems and addition and subtraction operations, especially with fractions. For example, the number sentence "12½ − 10½ = ?" can be modeled on the ruler as starting at 12½ and moving a distance of 10½ units to the left.

Method

PARTICIPANTS

Participants were 63 sixth graders who participated in a PLM instructional intervention, plus 49 seventh graders and 29 eighth graders who served as uninstructed control participants. All participants were enrolled in an urban public middle school serving a predominantly low-income neighborhood.

MATERIALS

The PLM software presented learners with a graphic display showing a ball on top of a ruler and a billiard cue poised to strike it. Learners were presented with four types of trial formats, which varied what information was given and what information was to be found (e.g., given the start point and endpoint, find the distance traveled; or given the start point and distance traveled, find the endpoint). On some trials the user entered responses by keying them in using an onscreen interface. On other trials the user entered a response by dragging a marker on the ruler to the desired point. Once the learner had entered his or her response and pressed a button labeled "strike," the billiard cue would carry out the event on the screen. Animated feedback was provided on each trial. On incorrect trials, the feedback illustrated the correct answer in a way that allowed the student to compare the response entered with a correct response.

The learning items in the database varied the types of values involved, whether the rulers were fully versus partially labeled, and whether they were partitioned in the most economical way to solve the problem or were overpartitioned (e.g., a ruler marked in units of one-sixteenth for a problem involving eighths). Movement on the ruler could be in either the rightward or leftward direction, though the endpoint was never a negative number. The quantities involved varied from single digits into the hundreds and included both fractions and integers. For larger values, the ruler did not start at zero but represented a segment of a ruler marked for numbers in the relevant range. Items in the learning set were classified into eight categories, including both fraction and integer problems.

DESIGN AND PROCEDURE

At the start of the study, sixth-grade students completed a 44-point pencil and paper assessment with a variety of items related to linear measurement with integers and fractions, and adding and subtracting fractions. A control group of seventh and eighth graders, who did not participate in any study-related instruction, were administered the same test just once, providing a baseline comparison for the sixth graders' scores.

After completing the pretest, sixth graders participated in a single introductory classroom lesson conducted by Zipora Roth. They then used the PLM software for a series of individual computer-based sessions. Students continued sessions with the software until they had either met the mastery criteria for all eight categories of problems in the PLM or completed six sessions. Within one to two days of completing their last computer session, students completed a post-test assessment. Four months later, the sixth graders completed a delayed post-test, with no study-related activities occurring in the interim.

PRETEST/POST-TEST ASSESSMENT

Participants were administered a 44-point pencil and paper assessment, with items distributed across a number of subscales. The subscales were designed to assess children's ability to use a partitioned number line to express the length of a line in generic units; to use both conventional and broken rulers to measure lengths in inches and centimeters; to use conventional and broken rulers to construct extents of varying lengths; to solve addition and subtraction problems with fractions; and to solve open-ended word problems involving linear measurements. Virtually all of the items were transfer items that did not directly resemble the trials presented to students during the PLM training. Three parallel forms of the assessment were created to allow equivalent tests to be administered to the same sixth graders in the intervention group as a pretest, an immediate post-test, and a delayed post-test. The forms were counterbalanced across children and times of administration, so that the child saw a different form each time the assessment was administered.

Results

As Figure 16.2 indicates, prior to instruction, the sixth graders and the seventh- and eighth-grade control groups scored similarly on the assessment, with no evidence of improvement through middle school. At the time of the immediate post-test, the sixth-grade intervention group showed significant improvement, as indicated by a one-way ANOVA comparing the sixth-, seventh-, and eighth-grade groups [$F(2, 138) = 19.687$, $p < .001$]. The sixth graders achieved nearly identical scores on a delayed post-test administered four months later, indicating that their learning gains were fully maintained. Amongst other subscales, the assessment included sets of items comparing students' performance reading and constructing lengths with conventional versus broken rulers. The sixth-grade intervention group improved on both types of items—but especially on the broken ruler items—from pretest to post-test. Students also made strong gains for items involving fractions.

Discussion

These results indicate that an intervention that consisted primarily of PLM training over the course of about half a dozen class periods yielded dramatic improvements in sixth-grade students' ability to demonstrate competence with linear measurement

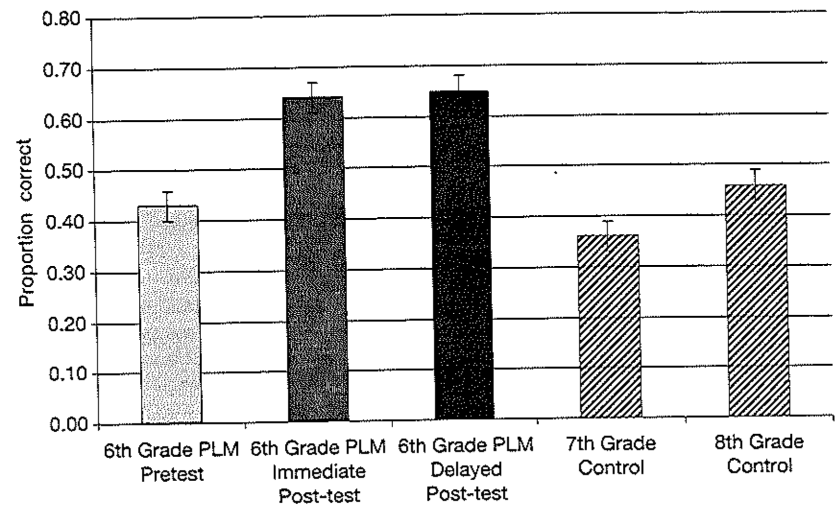

Figure 16.2 Pretest and Post-test Scores on Measurement and Fractions Assessment for Sixth Graders in Intervention Condition in Study 2 Compared with Seventh- and Eighth-Grade Control Groups. Error bars indicate ± one standard error of the mean.

involving both integers and fractions. These results are documented by robust and long-lasting gains on a quantitative assessment, but we also observed qualitative signs of changes in the children's performance. We think it is fair to state that many, perhaps most, of the students at the start of the study had little insight into the structure of linear units of measurement, their organization on a ruler, or the use of fractions to describe partitions of units. At the end of the study, many students had undergone a qualitative change in their understanding, such that they could, literally, perceive the structure and organization of measurement units in rulers and the relations between units and subunits. Their performance on transfer problems indicates that they could also interact with these units and their relations in productive and meaningful ways to solve new problems.

In contrast to the successful learning of the sixth graders in the study, the data obtained from the seventh- and eighth-grade control groups indicate that there otherwise appeared to be no significant learning in this area after two more years of middle-school math instruction. Persisting difficulties in understanding measurement and fractions leave students severely compromised in their ability to work with continuous quantities in meaningful ways, whether in or out of school, and it puts them at significant risk as they move on to more advanced mathematics in high school.

Conclusion

The research described in this chapter suggests that perceptual learning techniques offer clear promise for improving learning in mathematics and other domains. In fraction learning, instruction that focused on recognizing and mapping structural patterns led to clear improvements in students' solving of fraction problems. In measurement, PL interventions led to deep advances in learners' competence with fundamental notions of extent, units of measurement, and application of both integer and fractional quantities. The efficacy of PL techniques was shown in comparison to a no-PLM control in the fraction study, and was shown to boost learning beyond levels found even two grades later in the measurement study. Such results not only indicate that PL methods can succeed in notoriously difficult pedagogical domains but suggest that they do so because they address dimensions of learning that are poorly addressed by conventional instruction. Improved instruction in mathematics and other learning domains may require complementing the declarative and procedural emphases of classrooms with methods such as PLMs that advance students' structural insight and fluency.

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