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Object Interpolation in Three Dimensions

Philip J. Kellman and Patrick Garrigan
University of California, Los Angeles

Thomas F. Shipley
Temple University

Perception of objects in ordinary scenes requires interpolation processes connecting visible areas across spatial gaps. Most research has focused on 2-D displays, and models have been based on 2-D, orientation-sensitive units. The authors present a view of interpolation processes as intrinsically 3-D and producing representations of contours and surfaces spanning all 3 spatial dimensions. The authors propose a theory of 3-D relatability that indicates for a given edge which orientations and positions of other edges in 3 dimensions may be connected to it, and they summarize the empirical evidence for 3-D relatability. The theory unifies and illuminates a number of fundamental issues in object formation, including the identity hypothesis in visual completion, the relations of contour and surface processes, and the separation of local and global processing. The authors suggest that 3-D interpolation and 3-D relatability have major implications for computational and neural models of object perception.

Objects are fundamental units in human representations of reality. They serve as organizing elements of actions, thoughts, memories, and language. The value and centrality of object representations derive from their most basic purpose: to describe the functional units of the physical world. Object representations encode which things are connected entities and which are separate, which will function as units as people act on them and which will not. To these basic units people attach a host of important properties—shape, size, material composition, and function, among others.

Given objects' centrality, obtaining representations of them counts among the most important functions of perceptual systems. For this task, vision is preeminent, because information in reflected light furnishes the most detailed representations of objects. Few cognitive activities surpass the seeing of objects in ease, speed, and accuracy, yet the efficiency of this operation derives from processes of considerable complexity that, in many respects, remain mysterious.

The understanding of object perception is advancing. In recent years, theorists and experimenters have identified a number of computational tasks that contribute to this understanding, and they have proposed neural-style models of how these tasks may be accomplished (Barrow & Tenenbaum, 1986; Heitger, Rosenthaler, von der Heydt, Peterhans, & Kubler, 1992; Morrone & Burr, 1988). These models typically begin with known properties of orientation-sensitive and end-stopped cells in early visual cortical areas that are inputs to edge and junction detection (Heitger et al., 1992) and contour integration (Yen & Finkel, 1998).

Along with the detection of surface properties—including color, texture, and depth—these tasks form the departure point for further computations, including interpolation of contours and surfaces, formation of units, and description of their shapes (see Kellman, Guttman, & Wickens, 2001, for a framework relating various processing tasks in object perception). Although complex, these processes of visual segmentation and grouping have also become better understood in recent years. Research has improved understanding of the geometric relationships that support contour and surface interpolation (Fantoni & Gerbino, 2003; Geisler, Perry, Super, & Gallogly, 2001; Kellman & Shipley, 1991; Kellman, Yin, & Shipley, 1998), the constraints imposed by boundary assignment (Anderson & Julesz, 1995; Nakayama, Shimojo, & Silverman, 1989; Rubin, 2001; Shiffrar & Lorenceau, 1996), the contributions of symmetry (Sekuler, Palmer, & Flynn, 1994; van Lier, 1999), and the possible bases of these computations in the interactions of oriented units in visual cortex (Field, Hayes, & Hess, 1993; Heitger, von der Heydt, Peterhans, Rosenthaler, & Kubler, 1998; Thornber & Williams, 2000; for reviews, see Kellman et al., 2001; Neumann & Mingolla, 2001).

Most research on visual segmentation and grouping has focused on static, 2-D displays. Yet a number of considerations suggest that studying these phenomena in two dimensions offers a limited view of what are really 3-D perceptual tasks and processes. Obviously, objects and surfaces in the world extend in three spatial dimensions. More relevant to perceptual theory, evidence suggests that people's representations of objects, or at least some of them, encode 3-D information (e.g., Liu, Knill, & Kersten, 1995). As we consider below, many aspects of visual phenomena and processes that determine the connectivity and boundaries of regions indicate the importance of all three spatial dimensions. In short, both the inputs to visual segmentation and grouping processes and their outputs are likely to be inherently 3-D. Some research has begun to address the third dimension in visual object formation (e.g., Carman & Welch, 1992; Hakkinen, Liinasuo, Kojo, & Nyman, 1998; Heider, Spillman, & Peterhans, 2002; Kellman & Shipley, 1991). In the present work, we offer a systematic theoretical account of 3-D object formation, centered on the interpolation of contours across gaps in 3-D space.

The organization of the article is as follows. We first review a number of research findings that bear on 3-D aspects of visual

Philip J. Kellman and Patrick Garrigan, Department of Psychology, University of California, Los Angeles; Thomas F. Shipley, Department of Psychology, Temple University.

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Correspondence concerning this article should be addressed to Philip J. Kellman, Department of Psychology, 2291B FH, University of California, Los Angeles, Los Angeles, CA 90095. E-mail: kellman@psych.ucla.edu

segmentation and unit formation. To account for the geometry of 3-D object formation, we propose a theory of 3-D relatability. For a given edge, the theory specifies the 3-D orientations and positions of other visible edges that may be connected to it by visual interpolation. In related empirical work (Kellman, Garrigan, Shipley, Yin, & Machado, 2005), we have introduced a new, objective method to study 3-D interpolation processes. A series of experiments yielded objective evidence for 3-D interpolation and strong support for key aspects of the theory of 3-D relatability. We briefly describe those findings here. We then elaborate the theory of 3-D relatability and phenomena of 3-D interpolation in relation to several fundamental issues in object formation: the geometry of object formation, the identity hypothesis in object completion, the role of complementary contour and surface processes, global and local processes, and neural models of object formation. As we show, the theory and data regarding 3-D relatability suggest that current neural-style models of contour interpolation, based directly on 2-D-oriented units, cannot account for contour interpolation across all three spatial dimensions. Rather, neural units' coding position and orientation in 3-D space, such as those recently discovered in the caudal intraparietal sulcus (cIPS; Sakata, Taira, Kusunoki, Murata, & Tanaka, 1997), are likely candidates for the substrate of interpolation in object formation.

Background

What is known about these issues from prior research? The overall answer regarding 3-D interpolation is that there is little in the way of either systematic theory or objective experimental data. A number of investigators have explored important aspects of 3-D segmentation and grouping, and some of their results are relevant to our primary questions. We review some of this research below, categorizing it according to the 3-D aspects involved. Specifically, we discuss (a) 3-D aspects inherent in any occlusion display, (b) use of binocular disparity to produce depth stratification in illusory and occluded displays, (c) volume completion, and (d) prior work on 3-D illusory contours. We then summarize some aspects of object completion and contour relatability in two dimensions as relevant background for our treatment of 3-D contour interpolation.

2-D Interpolation Under Occlusion

Implicit in virtually all studies of occlusion is some stratification of surfaces in depth and, therefore, some 3-D component. Most studies have used static, 2-D figures that are partially occluded. In these displays, interpolation itself proceeds not in three dimensions but in a frontoparallel plane. Some research suggests that explicit manipulation of the depth of an occluding or intervening surface, using stereoscopic depth, influences the strength of object completion. Nakayama et al. (1989) manipulated the stereoscopic depth of a surface that interrupted the optical projection of 2-D face stimuli. Their data indicated better face recognition when the intervening surface appeared as an occluder (i.e., nearer to the observer than the face) than when it appeared as a recessed surface (i.e., farther away than the visible parts of the face). They argued that the depth position of the intervening surface acted as a gating influence on interpolation, probably because it altered the boundary ownership of the contours between the face parts and the intervening strip. (Interpolation was facilitated when the boundaries were *intrinsic* to the occluder—i.e., when the occluder "owned" these boundaries.) Other research has also explored gating influences of occluder depth on interpolation of frontoparallel image fragments (e.g., Anderson & Julesz, 1995; Brown & Koch, 1993).

Depth Stratification and Interpolation Based on Disparity Differences

T. Shipley (1965; see also Heider et al., 2002; Julesz, 1971; Poom, 2001) showed that illusory contours can form between edges created by disparity in random-dot stereograms. When disparity information placed two frontoparallel rectangles, separated by a gap, in front of their backgrounds, illusory contours formed across the blank area in between. The 3-D aspect here was the separation of background and object planes, although interpolation still involved connections between edges in a common frontoparallel plane. The fact that interpolation occurred despite the recessed blank area separating the two rectangles appears to be at variance with the findings of Nakayama et al. (1989). The difference may be that the monocular images contained visible texture in the intervening areas in Nakayama et al.'s experiments.

Several researchers (Anderson & Julesz, 1995; Gillam, 1995; Liu, Stevenson, & Schor, 1994, 1995; Mustillo & Fox, 1986; Nakayama & Shimojo, 1990) have shown that illusory contours can be formed even from half-occluded features (i.e., features that project only to one eye). The presence of a feature in one eye and its absence in the other can be interpreted by the visual system as the presence of an occluding edge. These loci can be used to induce illusory contours of an occluding surface.

Volume Completion

Some research (Tse, 1999a, 1999b) has examined 2-D displays that are claimed to evoke representations of volumes. On the basis of subjective report data using these displays, it has been argued that the 2-D displays produce representations of volumes, which in turn have unique effects on interpolation. This suggestion is intriguing, but there is little evidence for it. Tse (1999b) argued for volume completion on the basis of the idea that completion in certain displays would not be predicted by contour interpolation (and, specifically, not by the contour-relatability geometry proposed by Kellman & Shipley, 1991). Although completions in Tse's displays would not be predicted by contour relatability, they are straightforwardly predicted by the surface-spreading process that complements contour interpolation (Kellman & Shipley, 1991; Yin, Kellman, & Shipley, 1997, 2000). It is difficult to assess the possibility of volume completion from data that are predicted by an established surface-completion process that is known to operate in situations in which volumetric perception is not a possibility. (For further discussion of the application of surface spreading to so-called volume-completion displays, see Kellman et al., 2001.) There might be ways to distinguish true volume completion from surface spreading. Unfortunately, existing results used to invoke interpretations of volume completion have all relied on 2-D displays and subjective report methods. No evidence indicates objectively that these displays were seen as volumes or, if they were, that the completion processes depended on such representations.

3-D Illusory Contours

Although research in each of the categories considered above has involved the third dimension in some way, none of it has directly addressed interpolation processes using as inputs 3-D positions and orientations of edges and producing as outputs interpolated contours and surfaces extending through all three spatial dimensions. These issues are our main focus. Some demonstrations and a few experiments have addressed these issues directly, suggesting that object completion might be a 3-D process. Kellman and Shipley (1991) presented a demonstration of 3-D interpolation, using 3-D positions and orientations of inducing elements to produce interpolated contours and surfaces extending through all three spatial dimensions. A display similar to theirs is shown in Figure 1. Using stereoscopic disparity, the display induces for a single object both a 3-D illusory contour and a 3-D completion of a partly occluded portion. Consistent with the identity hypothesis in contour interpolation (see Kellman et al., 1998; T. F. Shipley & Kellman, 1992a), these were hypothesized to derive from the same interpolation process, with the appearance as illusory or occluded contours depending on other surfaces and depth information in the scene. Accordingly, reversal of the views given to the left and right eyes switches which portion of the object appears as partly occluded and which appears as a 3-D illusory surface. Kellman and Shipley conjectured that 3-D interpolation might depend on a 3-D generalization of contour relatability, but they did not elaborate.

Carman and Welch (1992) produced 3-D illusory contours and surfaces of several different shapes. They argued that reports in stereoscopic illusory-contour displays of different shape classes (Carman & Welch, 1992) or reports of systematic underperception of curvature (Vreven & Welch, 2001) indicate interpolated contours and surfaces.

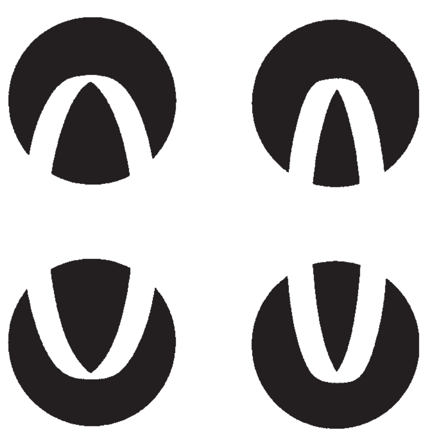

Figure 1. 3-D illusory and occluded contours. This stereo pair can be free-fused by crossing or diverging the eyes. When fused, the display produces the perception of a ring turned out of the picture plane. On one side (the left if the figure is cross-fused) is a 3-D illusory contour (i.e., a contour that extends through all three spatial dimensions). On the other side (the right if the figure is cross-fused) is a 3-D occluded contour. Switching left- and right-eye images also switches which contours appear illusory or occluded. From "A Theory of Visual Interpolation in Object Perception," by P. J. Kellman and T. F. Shipley, 1991, *Cognitive Psychology, 23,* p. 182. Copyright 1991 by Elsevier. Adapted with permission.

Previous studies or demonstrations have not systematically investigated the conditions under which 3-D interpolation occurs and fails to occur. Moreover, apart from brief reference to the possibility of a 3-D notion of contour relatability (Kellman, 2000; Singh & Hoffman, 1999a), there has been little development of theory about the conditions under which 3-D interpolation occurs. Some findings in computational vision regarding surface reconstruction from sparse stereoscopic information (Grimson, 1981; Weiss, 1990) bear important relationships to the contour relations we describe below. The relation between contour and surface processes in 3-D interpolation is taken up in the Contour and Surface Processes in 3-D Interpolation section below.

What are the conditions that initiate 3-D contour interpolation? What relations among visible contours support their connection by 3-D interpolation? The theory proposed here, and related empirical efforts (Kellman et al., 2005; Kellman, Machado, Shipley, & Li, 1996), address these questions.

Object Completion and Contour Relatability

What visible areas belong to the same objects or surfaces in the world? In ordinary visual environments, because of occlusion, the question can be highly complicated. Adjacent areas in the optical projection often belong to different objects or surfaces, whereas a single object may project to multiple, spatially separated visible regions (see Figure 2). In recent years, much has been learned about both the initiating conditions and the geometric relationships underlying object interpolation in two dimensions. Because some of this work provides the foundation for our proposals about 3-D interpolation, we review it briefly here.

Object interpolation depends on complementary processes of contour and surface interpolation (Grossberg & Mingolla, 1985; Kellman & Shipley, 1991). Contour interpolation plays the lead role in establishing shape. Surface properties spread under occlusion unless stopped by real or interpolated contours, and they are also confined by edge extensions under occlusion even when these are not connected to others (Yin et al., 1997, 2000). In this brief review, we emphasize contour processes, although both processes may be relevant to 3-D interpolation.

Contour interpolation connects image fragments across gaps. The most pervasive cause of gaps in the projections to the eyes is occlusion of parts of an object by other objects and surfaces. Because occlusion is so common in ordinary 3-D environments, perceiving most objects requires some application of contour-completion processes. Contour-interpolation processes also produce illusory contours, in which the interpolated contours appear in front of other surfaces in a scene. This situation is less common in ordinary environments because it depends on the relatively rare occurrence of an object's edge being imaged against a background so closely matched in color and brightness that there is no physical gradient visible across the contour. Although the cases of contour completion under occlusion (*amodal completion*, in the terminology of Michotte, Thines, & Crabbe, 1964) and illusory contours (*modal completion*) involve differences in the perceived position of interpolated contours relative to other visible surfaces, a variety of considerations and experimental evidence indicate that a single interpolation process contributes to both (Kellman et al., 1998; T. F. Shipley & Kellman, 1992a). The initiating conditions for this process are junctions or corners in visible contours—formally,

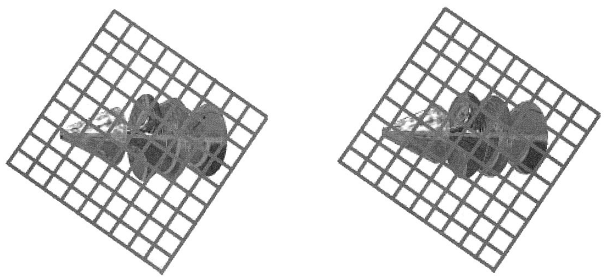

Figure 2. Example of a multiply occluded object. This stereo pair can be free-fused by crossing the eyes. As is common in ordinary scenes, occlusion causes the object to project to each retina in multiple, spatially separated regions. Human perceivers have little difficulty recovering the unity and shape of objects under such conditions.

tangent discontinuities—locations at which contours have no unique orientation. T. F. Shipley and Kellman (1990) observed that in general, interpolated contours begin and end at these points in images and showed that their removal eliminated or markedly reduced contour interpolation. Heitger et al. (1992) termed these *key points* and proposed a neurally plausible model for their extraction from images.

Tangent discontinuities are necessary but not sufficient conditions for contour interpolation. After all, many corners in images are corners of objects, not points at which some contour passes behind an intervening surface (or in front, as in illusory contours). The remaining conditions for contour completion involve several factors. The most important involves the relative positions and orientations of pairs of edges leading into points of tangent discontinuity. Certain geometric relations between edges leading into points of tangent discontinuity are crucial for contour interpolation in 2-D cases. These relations have been described formally in terms of contour relatability (Kellman & Shipley, 1991; Singh & Hoffman, 1999a; cf. Geisler et al., 2001). The key idea in contour relatability is that the visual system interpolates contours that meet certain criteria. These include smoothness (e.g., interpolated contours are differentiable at least once), monotonicity (interpolated contours bend in only one direction), and (roughly) a 90° limit (interpolated contours bend through no more than 90°). Figure 3 shows a construction that is useful in defining contour relatability. Formally, if E_1 and E_2 are surface edges, and R and r are perpendicular to these edges at points of tangent discontinuity, then E_1 and E_2 and are relatable if and only if

$$0 \leq R \cos\theta \leq r, R \geq r.$$

One Object or Two?

The relatability criterion given above defines the limits of relatability—that is, a range of positions and orientations of edges that allow the edges to be perceived as connected. Limits are important, because object perception often involves discrete categorization. Visual processes must decide whether two visible fragments are part of the same object or not. This classification of visible parts may be decisive in determining whether certain kinds of further processing will occur (e.g., generation of a representation of areas as connected, generation of a single shape description encompassing those parts, comparison with items or categories in memory, etc.). Recent research indicates that the representation of visual areas as part of a single object or different objects has a variety of important effects on information processing (Baylis & Driver, 1993; Zemel, Behrmann, Mozer, & Bavelier, 2002).

Quantitative Variation

Although the discrete classification of visible areas as connected or separate is important, there is also reason to believe that quantitative variation exists within the category of relatable edges (Banton & Levi, 1992; Field et al., 1993; Kellman & Shipley, 1991; T. F. Shipley & Kellman, 1992a, 1992b; Singh & Hoffman, 1999b). Factors affecting the strength of interpolation include support ratio (Banton & Levi, 1992; T. F. Shipley & Kellman, 1992b), relative angle of relatable edges (Field et al., 1993), and misalignment (T. F. Shipley & Kellman, 1992a). Figure 4 illustrates variation with angular deviations from collinearity. It shows a series of edge pairs with angular relations varying from 180° (collinear) to more than 90° (approximate limit of relatability). Experiments indicating a decline to a limit around 90° were reported by Field et al. (1993). Singh and Hoffman (1999a) proposed an expression for quantitative decline of relatability with angular change.

Ecological Foundations

The notion of relatability is sometimes described as a formalization of the Gestalt principle of *good continuation* (Wertheimer, 1923/1938). Recent work suggests that good continuation and relatability are separate but related principles of perceptual organization (Kellman, Garrigan, Kalar, & Shipley, 2003). Both embody underlying assumptions about contour smoothness (Marr, 1982), but they take different inputs and have different constraints. The smoothness assumptions related to both of these principles reflect important aspects of the physical world as it projects to the eyes. Studies of image statistics suggest that these principles approach optimality in matching the structure of actual contours in

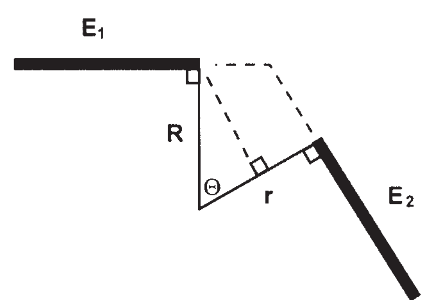

Figure 3. Geometric definition of 2-D relatability. Two edges (E_1 and E_2) are relatable if and only if the two perpendiculars R and r (chosen so that $R \geq r$) extending from the ends (tangent discontinuities) of E_1 and E_2 meet and the angle between R and r, θ, is bounded by the relationship $0 \leq R \cos\theta \leq r$. From "3-D Interpolation in Object Perception: Evidence From an Objective Performance Paradigm," by P. J. Kellman, P. Garrigan, T. F. Shipley, C. Yin, and L. Machado, 2005, *Journal of Experimental Psychology: Human Perception and Performance, 31,* p. 560. Copyright 2005 by the American Psychological Association. Reprinted with permission.

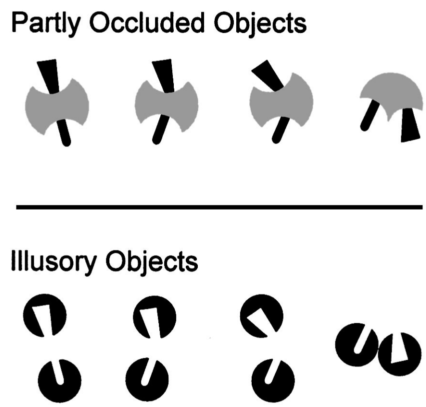

Figure 4. Variations in relatability with angular deviation from collinearity. From "3-D Interpolation in Object Perception: Evidence From an Objective Performance Paradigm," by P. J. Kellman, P. Garrigan, T. F. Shipley, C. Yin, and L. Machado, 2005, *Journal of Experimental Psychology: Human Perception and Performance, 31,* p. 560. Copyright 2005 by the American Psychological Association. Reprinted with permission.

the world. Through an analysis of contour relationships in natural images, Geisler et al. (2001) found that the statistical regularities governing the probability of two edge elements co-occurring correlate highly with the geometry of relatability. Two visible edge segments associated with the same contour meet the mathematical relatability criterion far more often than not. The success of relatability in describing perceptual interpolation processes (Field et al., 1993; Kellman & Shipley, 1991) may derive from ecological regularities in the natural environment.

Theory of 3-D Relatability

The foregoing considerations lead us to a key question about object interpolation in three dimensions. What geometry governs the 3-D formation of visible fragments into objects? In the 2-D case, contour interpolation is the primary process that establishes units and their shapes. (The complementary surface-spreading process connects visible surface patches but does not by itself produce well-defined shape [see Yin et al., 1997].) Accordingly, we focus on contour relationships in 3-D object formation. We propose that contour interpolation in three dimensions is governed by a 3-D generalization of the notion of contour relatability (Kellman et al., 1996). In this section, we describe and discuss this notion of 3-D relatability.

Although relatability in two dimensions has a specific formal definition for edges in an image plane, it embodies several principles that may be three-dimensionally recast. Relatable edges are smooth (differentiable at least once) and monotonic (singly inflected). They match the orientations of physically given edges where they meet, at the endpoints (tangent discontinuities) of those edges, and they bend through no more than about 90°. With the exception of the last constraint, the conditions can be summarized by saying that the linear extensions of two relatable edges meet in their extended regions (Kellman & Shipley, 1991). (It is important to note, however, that the meeting of linear extensions does not define the form of edges actually seen; interpolations are in general smooth, having no sharp corners.) The geometry defining relatability is naturally compatible with a particular form for contour connections: Every allowable interpolation could be composed of a straight (zero-curvature) segment and/or a constant-curvature segment (for details, see Kellman & Shipley, 1991, Appendix B; cf. Ullman, 1976). However, other forms for curved connections have also been proposed (e.g., Fantoni & Gerbino, 2003). The exact form of smooth, interpolated contours has proven difficult to determine, because many models make very similar predictions in many circumstances. Earlier work has looked at the shape of surface interpolation in structure-from-motion displays (Saidpour, Braunstein, & Hoffman, 1994). Saidpour et al. found that the shapes of interpolated surfaces were smooth and dependent on the geometry of the motion-defined surface, consistent with 3-D extensions of the models proposed by Ullman (1976) and Kellman and Shipley (1991) or a 3-D model of shape interpolation based on minimization of the quadratic variation functional (Grimson, 1981). However, in contrast to the issue of the exact form of interpolated contours, there is considerable evidence about the conditions under which 2-D interpolation does or does not occur (e.g., Field et al., 1993; Kellman et al., 1998; Ringach & Shapley, 1996).

A 3-D version of relatability would hold that within some threshold tolerance, the linear extensions of edges oriented in 3-D space must meet in their extended regions in 3-D space. An equivalent way of expressing this constraint is that relatable edges must be (roughly) coplanar (in any plane, not necessarily a frontoparallel plane), and within the plane that they define, they must meet the 2-D-relatability criteria. Figure 5 shows some examples of edge positions and orientations that meet these criteria. The set of relatable orientation and position combinations might be called the 3-D *relatability field.*

Expressing the limits of the 3-D relatability field formally can be done as follows. Consider two edges E_1 and E_2 having 3-D direction vectors **a** and **b** and ending at tangent discontinuities at points u_1 and u_2 (in R^3). Whether their linear extensions meet at all (i.e., whether they define a plane) is given by the minimum distance between the two lines d, where

$$d = \frac{|(u_2 - u_1) \cdot (\mathbf{a} \times \mathbf{b})|}{\|(\mathbf{a} \times \mathbf{b})\|}.$$

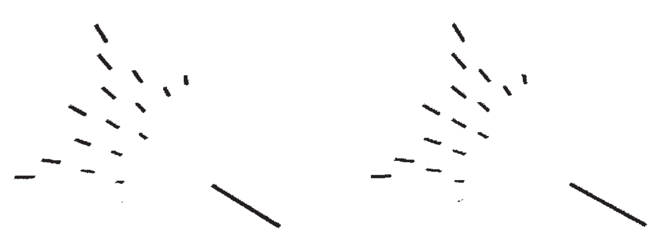

Figure 5. An example of a 3-D relatability field. This stereo pair can be free-fused by crossing the eyes. For a given surface edge in 3-D space, relatability of other edges depends on both their 3-D positions and orientations. From "3-D Interpolation in Object Perception: Evidence From an Objective Performance Paradigm," by P. J. Kellman, P. Garrigan, T. F. Shipley, C. Yin, and L. Machado, 2005, *Journal of Experimental Psychology: Human Perception and Performance, 31,* p. 561. Copyright 2005 by the American Psychological Association. Reprinted with permission.

When $d = 0$, the two edges define a plane. We would expect some small tolerance, γ, expressing the maximum deviation from coplanarity for which edge interpolation still occurs (i.e., when $d \leq \gamma$).

For two edges with $d \leq \gamma$, within their common plane, if R and r are their perpendiculars, meeting in an angle θ, they are relatable if and only if

$$0 \leq R \cos\theta \leq r.$$

The latter formulation is perhaps a simple way to express 3-D relatability, but it is probably not indicative of how 3-D relatability might be computed in neural machinery. A more plausible route might be to think of each visible edge ending at a contour junction (tangent discontinuity or key point) as initiating edge activation that extends out from that point. This activation can be described as a vector field, in which activation is assigned to other possible linking edges, depending on their 3-D locations and orientations. In other words, interpolation may depend on spreading activation in a field of possible linking edges (Field et al., 1993; Yen & Finkel, 1998). Neurally, the propagation of these activations may be carried out by the kinds of lateral interactions known to exist among orientation-sensitive units in visual cortex (e.g., Gilbert & Wiesel, 1985, 1989; Kapadia, Gilbert, & Westheimer, 1995).

Describing 3-D relatability in this manner, we define for a given edge and any arbitrary point the range of orientations at that point that fall within the limits of relatability. In the Cartesian coordinate system, we specify two angles, θ and φ. θ gives the orientation in the x–y plane, with 0 defined as oriented parallel to the x-axis. φ gives the orientation in the x–z plane, with 0 also defined as oriented parallel to the x-axis. For convenience, we place one edge so that its tip ends at the origin of the coordinate system (0, 0, 0), with an orientation $\theta = \varphi = 0$. For an edge terminating at some other point ($x \geq 0$, y, z), the range of possible orientations (θ, φ) for 3-D-relatable edges terminating at that point is given by

$$\tan^{-1}\left(\frac{y}{x}\right) \leq \theta \leq \frac{\pi}{2},$$

and

$$\tan^{-1}\left(\frac{z}{x}\right) \leq \varphi \leq \frac{\pi}{2}.$$

The lower bounds of these equations express the absolute orientation difference (−180° for two collinear edges ending in opposite directions) between the reference edge (edge at the origin) and an edge ending at the arbitrary point oriented so that its linear extension intersects the tip of the reference edge. The upper bounds incorporate the 90° constraint in three dimensions.

This formulation of 3-D relatability specifies its hypothesized limits; it does not define quantitative variation within those limits. That is, it does not yet assign variable strengths of interpolation depending on position and orientation of edges within the limits of relatability. Below, we discuss the issue of quantitative variation in interpolative strength after we summarize experimental results regarding the applicability and usefulness of 3-D relatability.

The idea of a 3-D association field governing 3-D interpolation retains some basic intuitions from previous theories based on 2-D interactions. At the same time, it marks a major departure from these models. Neural-style models of contour linking in object formation have almost all been based on known units in early visual cortical areas (V1 and V2) that signal 2-D orientation at particular locations. A great deal of recent research has sought physiological evidence for the loci of interpolation (Bakin, Nakayama, & Gilbert, 2000; Mendola, Dale, Fishl, Liu, & Tootell, 1999; Seghier, Dojat, Delon-Martin, Rubin, & Warnking, 2001; Sugita, 1999) and neural interactions (Murray, Wylie, Higgins, Javitt, & Schroeder, 2002; Peterhans & von der Heydt, 1989; Polat & Sagi, 1994) that may proceed there. As we argue below, both the basic requirements of 3-D relatability and the evidence of 3-D interpolation in human vision suggest that the earliest visual cortical areas may be insufficient to accomplish interpolation. Yet the basic insight of interpolation being accomplished through interactions of activated, orientation-sensitive units may be correct. For 3-D object formation, however, such interactions must involve units that signal the orientations and positions of edge fragments in 3-D space.

How neurally plausible is the notion of a network of interacting oriented units in three dimensions? Although no definite neural locus has been identified, recent research using single-cell recording in the cIPS indicates the presence of units tuned to particular 3-D slants (Sakata et al., 1997). It is notable that these units appear to respond to a particular 3-D orientation regardless of whether that orientation is specified by stereoscopic information or texture information. These findings indicate where the kinds of input units required for 3-D relatability—namely, units signaling 3-D orientation and position—may exist in the nervous system.

Experimental Evidence for 3-D Interpolation and 3-D Relatability

Many phenomena of perceptual segmentation and grouping were conveyed initially through examples and demonstrations (e.g., Kanizsa, 1979; Michotte et al., 1964; Wertheimer, 1923/1938) and later through systematic perceptual report procedures, such as magnitude estimation (e.g., Day & Kasperczyk, 1983; Dumais & Bradley, 1976; T. F. Shipley & Kellman, 1992b). Both demonstrations and perceptual-report measures are useful, but they are limited by demand characteristics and by the influences of cognitive strategies in addition to the perceptual processes they aim to assess.

In recent years, objective performance methods have become standard in investigating perceptual organization (e.g., Guttman, Sekuler, & Kellman, 2003; Ringach & Shapley, 1996; Sekuler et al., 1994). Until recently (Kellman et al., 2005), we know of no studies in which 3-D interpolation has been assessed using such an objective performance paradigm. In fact, there have been only a few experimental studies of any sort—notably by Welch and colleagues (Carman & Welch, 1992; Vreven & Welch, 2001). Carman and Welch summarized free reports of 3 naive and 2 nonnaive observers who were shown 3-D illusory contour displays that were expected to produce several different surface forms (planar, parabolic, elliptic, or hyperbolic). The authors reported that all subjects spontaneously identified the correct shapes in the four displays.

Kellman et al. (2005) developed an objective performance method to study 3-D interpolation and used it to assess the theory of 3-D relatability. In this section, we summarize their results. (For more extensive discussion of the importance of objective performance methods in this domain and details of the experiments and results, see Kellman et al., 2005.)

The paradigm for testing 3-D interpolation was in some ways analogous to a useful method for studying 2-D illusory- and occluded-contour perception—the *fat–thin* method devised by Ringach and Shapley (1996). In this method, the observer makes a forced choice as to whether a display is wider at its middle than at its top and bottom (fat) or is thinner in the middle (thin). These configurations are obtained from a standard Kanizsa square by rotating the partial circle-inducing elements about their central axes. The difficulty of the task can be modulated by varying the amount of rotation of the elements. This task has been shown to be highly sensitive to interpolation of the contours between the inducing elements. Specifically, when contour completion occurs, accuracy (Ringach & Shapley, 1996) and speed of classification (Kellman et al., 1998) are better than they are with displays that do not support interpolation but have similar physical elements. Recent results using an image-classification paradigm (Beard & Ahumada, 1998; Gold, Murray, Bennett, & Sekuler, 2000) have provided direct evidence that the interpolated regions are used by subjects in this task.

The paradigm used by Kellman et al. (2005) for studying 3-D interpolation can be understood by viewing Figure 6. Each panel contains a stereo pair that can be free-fused by crossing the eyes. When a pair is fused, it is apparent that the cutout areas in the black inducing elements produce two white surface tabs slanted in depth in each display. In the two cases in the left column, these two tabs meet the criteria for 3-D relatability. Phenomenologically, the two tabs in each of these cases merge into a single 3-D illusory surface that traverses the gap in between. The appearance of two separate tabs can be seen in the right column. These displays do not meet the criteria of 3-D relatability; relative to the displays in the left column, one of the tabs has been shifted (translated) in depth to disrupt 3-D relatability. In each of these cases, the two white tabs each appear to come to a vague end—they do not connect.

An objective classification task can be defined using the rows of Figure 6. In the top row, note that the two tabs in each case lie in converging (intersecting) planes. In the bottom row, the two tabs in each display lie in parallel planes. (Parallel includes the coplanar case at the bottom left.) Note that this classification of converging versus parallel is independent of the issue of 3-D relatability (both relatable and nonrelatable stimuli can have converging or parallel tabs). Kellman et al. (2005) hypothesized that perception of the two tabs as part of a single, continuous object would facilitate comparison of the tabs. Subjects' task was to make a speeded, forced choice on each trial of whether the top and bottom tabs were converging or parallel. Nothing was said to subjects about illusory contours or surfaces. Kellman et al. (2005) hypothesized that analogous to the fat–thin task in two dimensions, 3-D contour interpolation in these displays would produce a performance advantage for 3-D-relatable displays over the nonrelatable displays.

Kellman et al. (2005) found that signal detection measures of sensitivity improved and response times were shorter in this classification task for 3-D relatable displays than for displays in which one piece was shifted so as to disrupt 3-D relatability. Figure 7 shows the sensitivity (d') effects for one experiment. In agreement

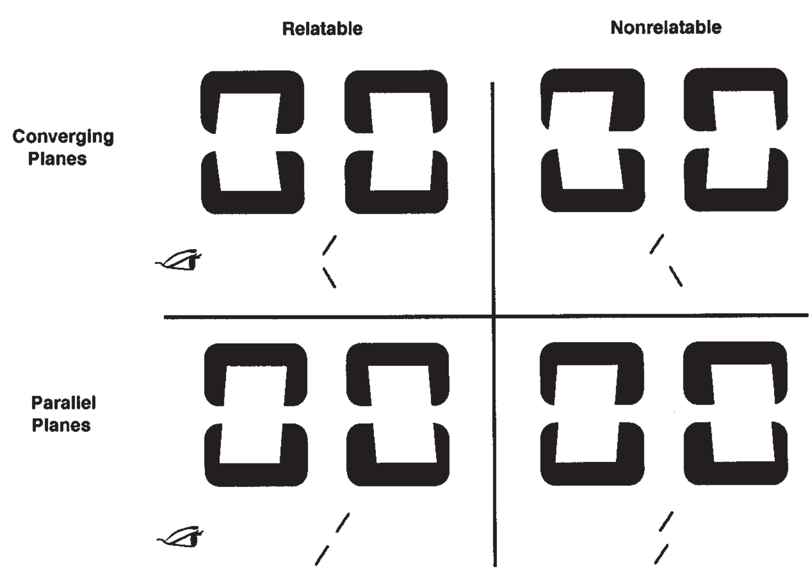

Figure 6. Examples of stimuli used in a 3-D classification task by Kellman et al. (2005). Each quadrant shows a stereo pair (which can be free-fused by crossing the eyes) in which two white regions oriented in depth are defined. For relatable displays, note the appearance of illusory contour and surface connections that form between the upper and lower regions. For nonrelatable displays, such connections are absent or indistinct. In the experimental task, subjects classified the two defined regions as being in converging or intersecting planes or in parallel planes. Note that this classification is independent of the status of displays as relatable or nonrelatable. (Side views showing depth relations to observer are depicted beneath each stereo pair.) From "3-D Interpolation in Object Perception: Evidence From an Objective Performance Paradigm," by P. J. Kellman, P. Garrigan, T. F. Shipley, C. Yin, and L. Machado, 2005, *Journal of Experimental Psychology: Human Perception and Performance, 31,* p. 562. Copyright 2005 by the American Psychological Association. Reprinted with permission.

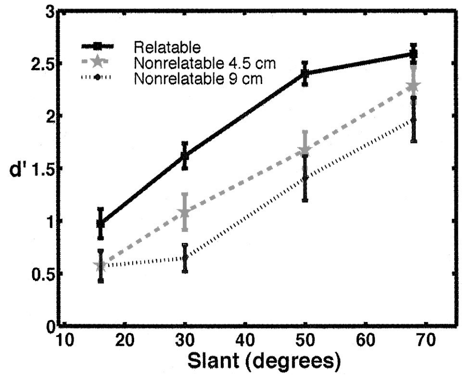

Figure 7. Sensitivity results from Experiment 1 in Kellman et al. (2005). Sensitivity (d') is shown for displays meeting the criteria of 3-D relatability and for two levels of nonrelatable displays, obtained by shifting one visible area in depth relative to the other. Errors bars represent plus or minus 1 standard error of the mean. From "3-D Interpolation in Object Perception: Evidence From an Objective Performance Paradigm," by P. J. Kellman, P. Garrigan, T. F. Shipley, C. Yin, and L. Machado, 2005, *Journal of Experimental Psychology: Human Perception and Performance, 31,* p. 565. Copyright 2005 by the American Psychological Association. Reprinted with permission.

with phenomenology, these results suggested that contour interpolation occurred in relatable but not in nonrelatable displays. Other results ruled out a number of alternative explanations. For example, small monocular misalignments involved in stereoscopic displays could not account for the effects, and the effects depended on the geometry of relatability rather than on alternative ideas about spatial relations that might have made comparisons easier (e.g., performance might be better for comparisons of tabs at similar depths).

In another experiment, using large lateral misalignments of inducing forms, it was found that contour interpolation, not surface spreading, accounted for better processing of relatable displays (Kellman, et al., 2005). This result (along with another) provided strong evidence that the role of 3-D relatability in these effects was its contribution to object formation. A manipulation that was expected to disrupt interpolation—rounding of tangent discontinuities—fully eliminated the superiority of relatable displays. This condition and the one with large lateral misalignments left the depth relations of tabs unchanged. Only the hypothesis that the effects depended on object formation predicted the disappearance of the clear advantage for 3-D-relatable displays when these additional requirements for interpolation were removed. A separate experiment investigated whether the observed interpolation effects violate a 90° constraint in 3-D interpolation and indicated that, because of slant underperception (see Gillam & Ryan, 1992), the 90°constraint was not violated.

The results of the experiments of Kellman et al. (2005) indicate that interpolation is a 3-D process that follows the geometry of 3-D relatability. Displays that met the constraints of 3-D relatability showed robust sensitivity and speed advantages over nonrelatable displays so long as other requirements for object formation, such as the presence of tangent discontinuities, were met but not otherwise. Below, we discuss the implications of these conclusions for the geometry of object formation and for several other crucial issues in computational and neural models of object perception.

3-D Relatability in Object Formation

The experimental evidence obtained by Kellman et al. (2005) regarding 3-D relatability supports several important conclusions about object perception. Contour-interpolation processes are 3-D in nature. Confirming earlier demonstrations and results using perceptual-report methods, Kellman et al.'s (2005) results provide objective evidence that object-formation processes take as inputs positions and orientations of edges in 3-D space and produce as outputs interpolated contours and surfaces that extend through all three spatial dimensions.

What is wholly new in the results of Kellman et al. (2005) is that they tested and found evidence supporting a particular geometric account of 3-D object formation. Specifically, the conditions under which 3-D interpolation occurs or fails to occur in their experiments were consistent with (and predicted by) the theory of 3-D relatability. Moreover, these recent empirical results support other key notions of the theory of 3-D contour interpolation, such as the importance of tangent discontinuities. By showing the interaction of 3-D relatability and other elements required for contour interpolation (e.g., tangent discontinuities and 2-D relationships), these recent results provide strong support for the present account of 3-D object formation.

This account places object formation on a new theoretical footing. Previous work describing the geometric relations that determine which visible fragments become connected into unitary objects had been largely concerned with 2-D relationships (Fantoni & Gerbino, 2003; Kellman et al., 2001; Kellman & Shipley, 1991). Likewise, neural models of these phenomena have almost universally utilized as inputs orientation-sensitive units that respond only to 2-D orientation (e.g., Grossberg & Mingolla, 1985; Heitger et al., 1998). The theory and results on 3-D object formation suggest that the 2-D phenomena, and the processes and mechanisms theorized to account for them, are special cases of a fundamentally 3-D process. The constraints on contour relations are not 2-D in nature but, rather, apply to all three spatial dimensions: Interpolated edges connecting across gaps in 3-D space must be smooth (differentiable at least once) and monotonic, agreeing with the orientations of real edges at their endpoints (points of tangent discontinuity). In this section, we provide further consideration of several aspects of this geometry. In the Neural Models, Computational Models, and 3-D Interpolation section below, we comment further on the implications for neural and computational models of the present theoretical account of the geometry of 3-D object formation.

Object Completion Geometry Versus Fortuitous Geometry

One important issue considered by Kellman et al. (2005) was whether results in their object-classification task were truly dependent on 3-D object formation. The most general alternative hypothesis that they considered was what they termed the *fortuitous-geometry hypothesis*. Perhaps the accuracy and speed advantages shown when subjects classified relatable displays were not due to object completion; it might be the case that relatability just happened to express geometric relations that made comparison of the two object parts on each trial easier than it was for nonrelatable displays. The primary way in which this explanation was ruled out involved manipulating other factors that should affect object completion while leaving the overall 3-D geometry unchanged. In one experiment, Kellman et al. (2005) used large 2-D misalignments of edges (which would be sufficient to disrupt 2-D completion) while preserving the relative depths and orientations of object parts. This manipulation removed the performance advantage for relatable displays. In another experiment, the exact 3-D geometry of relatable displays was preserved, but the tangent discontinuities (contour intersections) were rounded, a manipulation known to reduce or eliminate contour interpolation (T. F. Shipley & Kellman, 1990). The results unequivocally supported the notion that performance advantages observed for relatable displays depended on object completion, not merely a particular 3-D geometry that allowed easy comparisons of separate parts.

Misalignment in Depth

The experimental results of Kellman et al. (2005) supported the theory of 3-D relatability by showing that 3-D object formation, as predicted, is disrupted by introducing a depth shift of one contour relative to another. The depth shift must be of a magnitude large enough to place one of the contours outside of the limits of 3-D relatability described earlier. Connections between such contours would require a double inflection (to match the slopes of inducing contours at their endpoints) or the introduction of tangent discontinuities (e.g., if a linear connection formed between the ends of the visible edges). As in the 2-D case, 3-D contour-interpolation processes do not appear to create corners or to doubly inflect.

How much depth misalignment can be tolerated for shifts in depth? Kellman et al. (2005) found a difference between a depth shift of 5.6 arcmin of disparity and a larger shift of 11.2 arcmin. The larger shift apparently obliterated any interpolation effects, but a smaller shift of 5.6 arcmin did not completely eliminate them. Putting this in terms of positions in virtual space in the displays used in these experiments, relatability was reduced by a 4.5-cm shift (about a 5% shift at a viewing distance of roughly 95 cm) and eliminated by a 9.0-cm shift (about a 9% shift).

How best to quantify the small tolerance for shift cannot be decided from existing results. In the 2-D case, several empirical results suggest that interpolation is nearly eliminated by planar shifts of one contour by 15 arcmin of visual angle from an initially collinear or cocircular configuration (e.g., Palmer, Kellman, & Shipley, 1997; T. F. Shipley & Kellman, 1992a). However, a fixed retinal value for misalignment would not be viewpoint invariant. Whether two contour segments were relatable would vary with observer distance. This would be an ecologically odd arrangement and one that would differ from other known invariances in object perception, such as support ratio (Banton & Levi, 1992; T. F. Shipley & Kellman, 1992b). An alternative suggestion (Kellman et al., 2001) is that the tolerance depends on a ratio of lateral misalignment to contour separation (i.e., an angular notion). We are currently studying this issue in the 2-D case.

An angular metric might also be the relevant one in three dimensions. There may be several different ways to quantify it. Tolerance for relatability may decline with the ratio of depth shift (really 3-D shift in the general case) to a certain amount of contour separation. In the results of Kellman et al. (2005), this number would be rather large, on the order of 70°. In contrast, informal observations in our laboratory suggest that an angular constraint on 2-D misalignment would implicate angles (defined by horizontal misalignment and vertical separation) in the neighborhood of 17° as defining the borderline for interpolation (e.g., 15 arcmin of misalignment for displays with 50-arcmin vertical separation). Alternatively, 3-D misalignment might best be quantified by a ratio in which depth separation is given by a disparity difference (relative disparity) and compared with vertical separation (in visual angle). This measure would not be invariant with viewing distance (see below), because angular separation is inversely proportional to viewing distance, whereas disparity is (roughly) inversely proportional to the square of viewing distance. One problem with assessing the precise tolerance for depth misalignment is that depth, like slant, may not be veridically perceived in displays in which it is based solely on disparity. Tolerance for depth separation may be even less than Kellman et al. (2005) observed when position and orientation in depth are given by several correlated cues. Results obtained by these investigators indicated that both shift values substantially disrupted object interpolation; however, determining the most appropriate metric for quantifying depth shift remains a task for future research.

Tangent Discontinuities in 3-D Interpolation

An important basis for the claim that 3-D relatability describes the geometry of 3-D object formation is the effect of rounding of tangent discontinuities found by Kellman et al. (2005). This manipulation leaves the overall 3-D geometry of the visible tabs intact but disrupts another required ingredient for contour interpolation. Kellman et al. (2005) found that rounding of tangent discontinuities eliminated the difference between displays whose visible parts fit the geometry of 3-D relatability and displays whose visible parts did not.

These recent results are the first that we know of confirming the importance of tangent discontinuities as features that initiate object-completion processes in three dimensions. Along with other research (Guttman & Kellman, 2002; Palmer, Kellman, & Shipley, 2000; T. F. Shipley & Kellman, 1990) regarding tangent discontinuities, they suggest a unified, 3-D process of contour interpolation that is triggered by these important features in the optical projection.

Angular Limits of Interpolation

Another important question is whether interpolation in three dimensions has a cutoff when the relative angles between inducing contours are acute (as is approximately the case in 2-D interpolation). In the experiments of Kellman et al. (2005), the most extreme angles were acute. The data suggested robust 3-D completion effects even at the most acute slant value tested. On the face of it, these data raised questions about a 90° constraint in 3-D interpolation. However, another experiment (Kellman et al., 2005, Experiment 5) showed that slant was systematically underperceived in the displays, for reasons that were expected (see Kellman et al., 2005). Accordingly, Kellman et al.'s (2005) data could not be used to decide whether a 90° constraint governs 3-D interpolation—that is, whether reduction of the intersection angle beyond 90° would eliminate interpolation. This question is currently under investigation.

Torsion

Studies of 3-D interpolation have only recently begun to address the variable of torsion in 3-D relatability (Fantoni, Gerbino, & Kellman, 2004). For some point on a space curve parameterized by arclength, $\sigma(s)$, with principal normal vector **N** and tangent vector **T**, torsion, τ, is a scalar that defines how quickly the curve is bending out of the plane defined by **T** and **N**. Torsion can also be defined in terms of the change in the binormal curve, **B**, and **N**, at that same point (see Figure 8):

$$\mathbf{B} = \mathbf{T} \times \mathbf{N},$$

and

$$\frac{d\mathbf{B}}{ds} = -\tau\mathbf{N}.$$

At present, not much is known about tolerance for torsion. Some examples of displays involving torsion are shown in Figure 9. As the amount of twist required in an interpolated contour increases, the apparent connectedness between the top and bottom vertical edges decreases. There appear to be several separable cases to consider in assessing tolerance for torsion. In the displays in Figure 9, a connection between the two vertical edges of the partly occluded regions on either side of a display would require a curve with nonzero torsion. In the cases shown, however, the edges of the top and bottom are also differentially shifted in depth, as in the shifted conditions tested experimentally by Kellman et al. (2005). The lack of apparent interpolation in such displays might be a result of this shift. With regard to 3-D relatability, such displays violate the requirement that relatable edges be, within some threshold tolerance, coplanar. Other cases of torsion might not suffer from this confound. Figure 10 shows a display in which torsion is less confounded with shift. In this stereo display, the vertical edges are actually collinear. However, if such edges have an assigned orientation as part of the surfaces they bound, then the vertical edges above and below the occluders would have different binormal vectors. If this is so, an interpolated edge connecting them would have nonzero torsion (i.e., it would have to twist in going between the visible edges). Informal observation of this display and others in our laboratory suggests that there is very little tolerance for torsion in 3-D completion; however, this issue is currently being more systematically investigated (Fantoni et al., 2004).

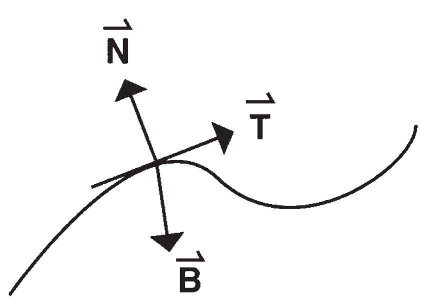

Figure 8. The principle normal (**N**), tangent (**T**), and binormal (**B**) of a space curve. From "3-D Interpolation in Object Perception: Evidence From an Objective Performance Paradigm," by P. J. Kellman, P. Garrigan, T. F. Shipley, C. Yin, and L. Machado, 2005, *Journal of Experimental Psychology: Human Perception and Performance, 31,* p. 580. Copyright 2005 by the American Psychological Association. Reprinted with permission.

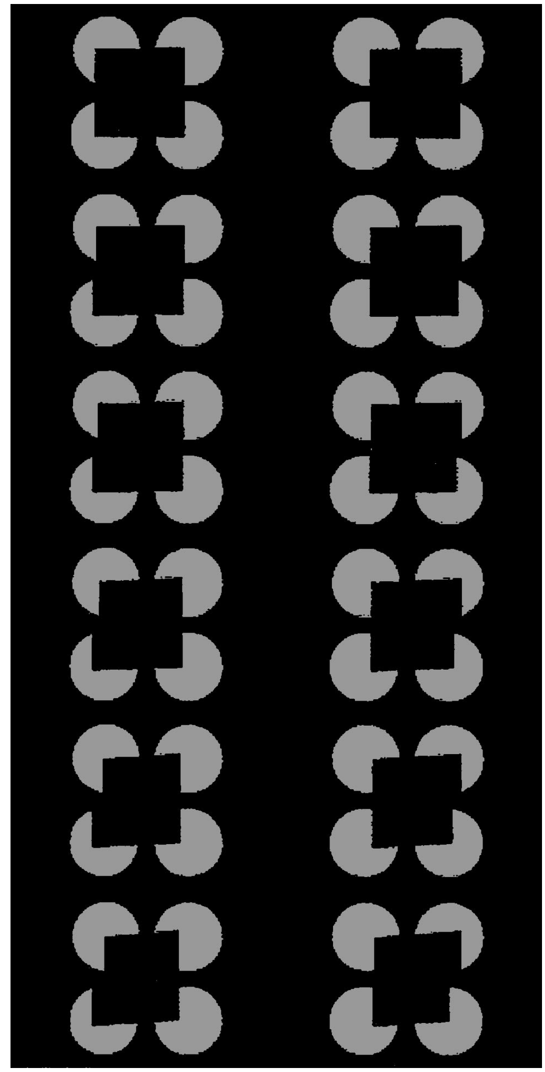

Figure 9. Interpolation displays requiring differing amounts of torsion. The displays are stereo pairs and can be free-fused by crossing the eyes. For interpolation to occur, the illusory contours must have torsion in all but the top display. The amount of torsion necessary for unit formation increases from top to bottom. From "3-D Interpolation in Object Perception: Evidence From an Objective Performance Paradigm," by P. J. Kellman, P. Garrigan, T. F. Shipley, C. Yin, and L. Machado, 2005, *Journal of Experimental Psychology: Human Perception and Performance, 31,* p. 580. Copyright 2005 by the American Psychological Association. Reprinted with permission.

3-D Relatability: Implications and Issues

In the preceding section, we considered several aspects of the geometry of object formation that relate to 3-D interpolation (and to 3-D relatability in particular). In this section, we consider more generally some of the implications of the theory of 3-D interpolation for object perception. In doing so, we describe some of the

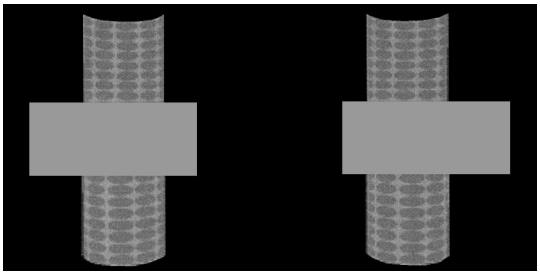

Figure 10. A separate case of torsion? The display is a stereo pair and can be free-fused by crossing the eyes. When the images are fused, the left and right vertical edges of the two half cylinders are collinear. However, the surfaces bounded by the top and bottom edges have different orientations. Contours have positions and orientations in space, but they may also carry an orientation derived from the surface they bound. If contour interpolation depends on all of these properties, these contours cannot be connected without substantial torsion. From "3-D Interpolation in Object Perception: Evidence From an Objective Performance Paradigm," by P. J. Kellman, P. Garrigan, T. F. Shipley, C. Yin, and L. Machado, 2005, *Journal of Experimental Psychology: Human Perception and Performance, 31,* p. 580. Copyright 2005 by the American Psychological Association. Reprinted with permission.

ways in which the theory helps to relate a number of findings in 2-D and 3-D object perception, the ways in which 3-D relatability mandates major changes in models of object perception, and some ways in which the theory may illuminate unresolved issues.

The Identity Hypothesis in 3-D Interpolation

We begin with an important issue regarding the scope of 3-D interpolation. On the basis of findings in two dimensions (Kellman et al., 1998) and spatiotemporal interpolation (Palmer et al., 1997), we believe that 3-D relatability applies to both occluded (amodal completion; Michotte et al., 1964) and illusory contours (modal completion). In the 3-D interpolation research summarized above, illusory-contour stimuli were used. This choice was motivated by several considerations (see Kellman et al., 2005, for details.)

3-D relatability appears to govern the appearance of connectedness in occluded displays, as can be seen in Figure 11. These displays are transforms—from illusory to occluded versions—of the displays shown in Figure 6. In the 3-D-relatable displays, the top and bottom tabs appear to connect into a single object behind the occluder. In the 3-D-nonrelatable displays, one tab has been shifted relative to the other. In these nonrelatable displays, the two tabs are seen as disconnected, with indeterminate endpoints behind the occluder.

In evaluating more thoroughly whether the identity hypothesis in contour interpolation applies to 3-D as well as 2-D cases, we may entertain weak or strong versions of the hypothesis. A weak version of the identity hypothesis would hold that the same geometry governs amodal and modal completion. A strong version of the hypothesis would hold that the actual process and the mechanisms of interpolating contours across gaps is common to both 3-D modal and amodal completion.

How might one decide whether to prefer the weak or the strong version of identity? In 2-D interpolation, a number of experimental results indicate similar determinants (Gold et al., 2000; T. F. Shipley & Kellman, 1992a), time course (Guttman & Kellman, 2004), and strength of interpolation (Kellman et al., 1998; Ringach & Shapley, 1996) in occluded and illusory displays that are equivalent (in that they have the same physically specified contours and gaps between them). Yet other research has suggested differences in constraints or processing (Anderson, Singh, & Fleming, 2002) or the neural substrate of amodal and modal completion (Corballis, Fendrich, Shapley, & Gazzaniga, 1999; von der Heydt, Peterhans, & Baumgartner, 1984). These apparent discrepancies have been discussed elsewhere (Kellman, 2003b; Kellman et al., 2001); in general, the difficulty in interpreting them is that they may not involve the interpolation process per se. There are clearly differences in the final scene representations and constraints for cases in which a completed object appears in front of other surfaces (modal completion) and cases in which a completed object appears partly behind another surface (amodal completion). (Otherwise, these displays could not look different.) In other words, not all aspects of amodal and modal displays, or of their processing, are identical, nor does the identity hypothesis claim that they are. Rather, the identity claim is that the specific process of generating contour connections across gaps is common to the two types of phenomena. The depth arrangements of final modal and amodal scene representations differ, and these differences depend on depth information and aspects of surface arrangement that differ in the stimulus (except in ambiguous cases that reverse between modal and amodal; see Kellman & Shipley, 1991). Hence, it is not surprising that some processing differences—for example, in the registration of visible areas (Ringach & Shapley, 1996)—can be found.

Conversely, the many results showing empirical similarities in processing cannot conclusively establish the strong identity hypothesis (Kellman et al., 1998). It can always be argued that interpolation is amodal and that modal completion is subject to similar determinants (geometry, etc.) yet involves distinct processes. In other words, it is imaginable that there could be a contour-connection process that operates for surfaces that appear in front of others (modal interpolation) and a separate contour-connection process that operates for surfaces that appear behind others (amodal interpolation).

Fortunately, there are compelling logical considerations (in connection with certain phenomena) that can be used to show that an identical contour-connection process underlies what observers ultimately experience as modal and amodal completion. Below, we consider these arguments and apply them specifically to 3-D interpolation.

The upshot of this discussion is that modal and amodal completion do not designate different processes but are different modes of appearance in final object and scene representations (Kellman, 2003a). The visual system marks with these importantly different modes whether contours and surfaces go behind other surfaces (relative to the observer's viewpoint) or not.

The notion that the process of interpolating contours across gaps is common to both modal and amodal cases rests on the logical requirements of two robust classes of phenomena: the existence of hybrid (or *quasimodal*) objects and the existence of phenomena in which contour interpolation clearly precedes determination of modal or amodal appearance. Below, we discuss each of these classes of phenomena in turn.

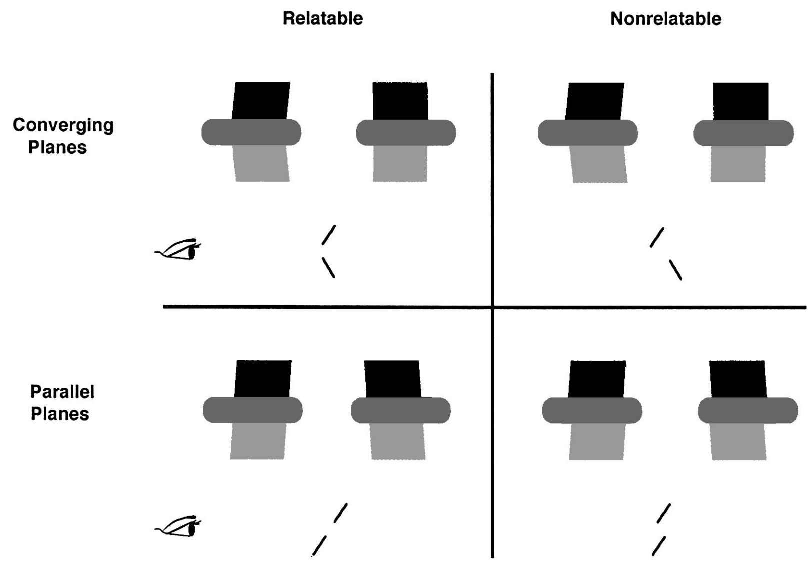

Figure 11. Amodal (partly occluded) displays equivalent to the modal (illusory) displays in Figure 6. Each quadrant shows a stereo pair (which can be free-fused by crossing the eyes) in which two planar tabs oriented in depth are defined. Side views showing depth relations to the observer are depicted beneath each stereo pair. Upper and lower tabs in each display differ in surface lightnesses to disrupt surface-spreading effects. For relatable displays, note that the top and bottom pieces appear connected behind the occluder. For nonrelatable displays, such connections are absent or indistinct.

Quasimodal objects. Kellman et al. (1998) showed that interpolated contours can occur in situations that fulfill neither the requirements of modal nor amodal completion. Interpolation of illusory contours ordinarily requires a pair of inducing elements each of which has an L junction at the points where an illusory contour joins a real contour. Interpolation of occluded contours ordinarily requires a pair of visible areas, each terminating in a T junction. Figure 12A shows an example of quasimodal objects (Kellman et al., 1998). In this display, robust contour interpolation is evident despite the fact that neither the requirements for illusory contour formation nor those for occluded contour formation are met. Moreover, Kellman et al. (1998) showed that quasimodal displays confer the same advantages and the same pattern of processing in an objective performance paradigm as do equivalent illusory and occluded displays. In Figure 12A, all of the interpolated contours have illusory contour-inducing elements (and an L junction) on one end of the interpolated contour and one inducer for an occluded contour (with a T junction) on the other. Figure 12B shows that these phenomena occur in 3-D interpolation: Every instance of contour connection in the display involves 3-D interpolation. Here again, the connected object is seen—with boundaries having definite positions in 3-D space—passing, in different places, in front of and behind other visible surfaces. Because none of the clear interpolated contours in these cases fulfills the requirements for so-called modal or amodal completion, must we now posit a third process—quasimodal completion? Note that this type of contour completion poses a paradox for claims that modal and amodal completion actually are processed in different cerebral hemispheres of the brain (Corballis et al., 1999) or in different neural areas (Peterhans & von der Heydt, 1989). There is, however, a rather simple explanation for quasimodal contours. The process that interpolates contours readily accepts relatable edges from either illusory or occluded contour inducers so long as these both terminate in tangent discontinuities and fit the geometry of relatability. Functionally, the importance of quasimodal interpolation is obvious: The visual system interpolates contours and surfaces. Sometimes, relative to the observer, the objects that the

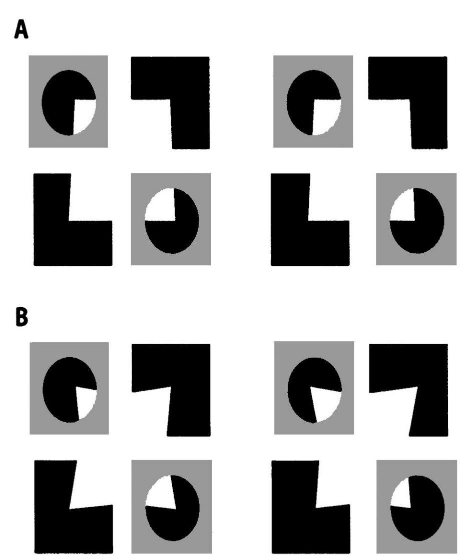

Figure 12. Examples of 2-D (A) and 3-D (B) quasimodal interpolation. The displays are stereo pairs that can be free-fused by crossing the eyes. In both, all interpolated contours connect illusory contour-inducing elements on one end and occluded contour inducers on the other.

visual system seeks to detect and represent are in front of other surfaces, and sometimes they are behind other surfaces. In a quasimodal case, a single contour is in front of some surface along part of its length and behind some surface along another part. That the edge passes behind another object in one location and in front of some object in another location is an additional fact worth representing (hence, the phenomenology of *modal* and *amodal*). But this fact does not imply that modal completion is one perceptual process and amodal completion is another. In fact, it seems clear that the process that connects contours across gaps readily incorporates segments of both kinds. It is not clear how quasimodal completions could occur from distinct modal and amodal interpolation processes.

Self-splitting objects and Petter's effect. The existence of quasimodal contours does not prove the identity hypothesis. It is conceivable, although hardly parsimonious, that there are three separate contour-completion processes: modal, amodal, and quasimodal. A more definitive logical argument about the nature of interpolative processes is implicit in phenomena that have been known for several decades. Consider the display in Figure 13A, an example of a class of displays studied by Petter (1956) and labeled *self-splitting objects* (SSOs) by Kellman and Shipley (1991). Although the SSO in Figure 13A is a contiguous, homogeneously colored and textured area, it is not seen as a single object. This splitting into two objects on the basis of tangent discontinuities and relatable edges is predicted by the object-formation model of Kellman and Shipley (1991). Perceptually, this display resolves into two: a triangle and a quadrilateral. Especially notable is that where parts of the two objects intersect, one object is seen as passing in front of the other. All parts of this display lie in the same depth plane, yet the visual system imposes a constraint that two objects may not occupy the same space; one must appear in front of the other.

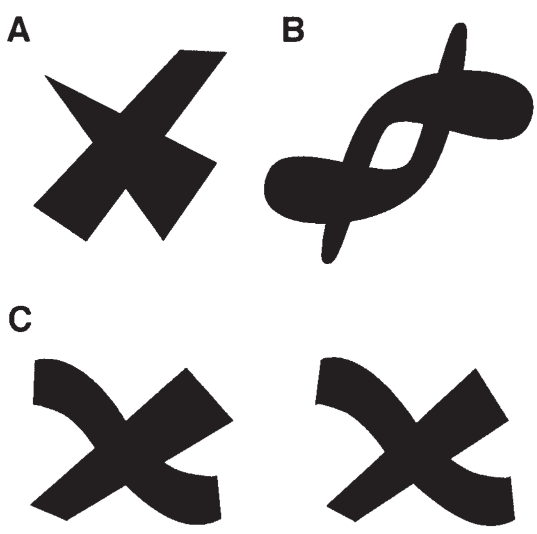

Figure 13. Self-splitting objects (SSOs) and their implications for the contour-interpolation identity hypothesis. A: A homogeneous region splits into two bounded objects; perceived depth order changes over time. B: An example of Petter's (1956) effect. The display splits into two objects with determinate depth order because of the relative extents of interpolated contours. C: A 3-D SSO display. The display is a stereo pair that can be free-fused by crossing the eyes. The ribbon-like object has illusory contours along parts of its boundaries and occluded contours along others.

Where the two objects cross, their physically specified boundaries have gaps; these gaps are spanned by interpolated edges. Because of the depth ordering, the nearer display has illusory contours, whereas the farther display has occluded contours. In Figure 13A, if the triangle appears in front, it has illusory contours, and the quadrilateral has occluded contours. However, the depth ordering of the two objects is unstable over time; which object appears in front fluctuates. When the triangle switches from being in front to being behind, its contours switch from being illusory to being occluded (and vice versa for the quadrilateral).

SSOs do not always possess this instability. The display in Figure 13B, similar to one devised by Petter (1956), appears more stable. It is seen as containing two interlocking, bent objects. Here again, the stimulus itself does not have luminance contours defining two objects; rather, the partitioning of the array into two bounded objects is the result of contour-interpolation processes.

The interlocking of the two perceived objects occurs because on each side, the thicker object passes in front of the thinner one. Petter (1956) noticed an interesting regularity in such displays, which can be stated as follows: Where interpolated boundaries cross, the boundary that traverses the smaller gap appears to be in front. Thus, the thicker parts of the objects appear in front of the thinner ones because the former have smaller gaps in their physically specified contours. Petter's rule also helps make sense of the reversibility of depth order—and of occluded and illusory contours—in Figure 13A: Both objects span roughly equal gaps; there is thus no consistent information about depth order.

As may now be apparent, Petter's (1956) effect has crucial implications for the identity hypothesis. According to Petter's rule, what determines whether an interpolated contour appears in front or behind—that is, as illusory or occluded (or modal vs. amodal)—is its length relative to the interpolated contours that it crosses. Logically, this statement implies some sort of comparison or competition involving the crossing interpolations. To accomplish this comparison, the visual system must first register the various sites of interpolation. Comparison of the lengths of the crossing interpolations precedes the determination of whether an interpolated contour ultimately will appear as illusory (in front) or occluded (behind). Therefore, the registration of interpolation sites and lengths must precede the determination of depth ordering. It follows that in at least some cases, contour-interpolation processes must operate prior to the processes that determine the final depth ordering of the constructed contours. This, in turn, implies that there cannot be separate mechanisms for the interpolation of contours in front of and behind other surfaces. At least the steps of locating sites of interpolation and the extents of interpolation must be in common.

SSOs and Petter's (1956) effect provide a compelling argument for a common underlying interpolation process. They are ordinarily illustrated (as in Figure 13B) with planar figures in a frontoparallel plane. Can the argument based on SSOs be directly extended to 3-D interpolation? As Figure 13C shows, it can. This SSO display is given as a stereo pair that can be free-fused by crossing the eyes. The display breaks into two objects, positioned in depth. The most interesting aspects of the display can be seen in the middle area, where no physically specified contours and no disparity information are given in the stimulus. The display appears stable, with the ribbon-like object passing through the planar quadrilateral (von Szily, 1921/1998). The ribbon-like object has illusory contours for part of its length—the part in which it passes in front of the quadrilateral. The remainder of its interpolated contours are occluded. This partly illusory, partly occluded status of the interpolated contours makes this a case of quasimodal interpolation, as described above. In complementary fashion, the quadrilateral has one interpolated edge that appears as modally completed (its upper edge) and one that is amodally completed (its lower edge). What determines the modal or amodal status of the various contours? As in Petter's effect, the modal or amodal status of each interpolated contour seems to depend on the status of the crossing interpolated contour. Unlike in Petter's effect, that status is not given by the size of the gap traversed. Instead, the status (which differs at different locations along a single contour) depends on the relative depth of points along the crossing interpolated contours. But given that the stimulus contains no contours and no local disparity information in the interpolated regions, where does relative depth come from? The answer, explained in the next section, provides a compelling argument for a common interpolation process in 3-D object formation.

Depth spreading and object formation. Figure 14 shows another type of display that implicates an interpolation process that is initially neither modal nor amodal (Kellman, 2003a; Kellman et al., 1998). As in Petter-effect displays, the reason is that interpolation must occur prior to the determination of modal or amodal appearance. The display shows a stereo pair that can be free-fused by crossing or diverging the eyes. (For the present analysis, we will assume that the reader free-fuses by crossing the eyes; if fusion is accomplished by diverging, all of the depth relations will be opposite to those in the following description, but the main effect will still be obvious.)

When fused, the central rectangle appears as the nearest object in the display at its left edge and as the farthest at its right. With reference to the two white vertical "columns" in the display, the rectangle appears to pass in front of the one on the left and behind the one on the right. This simple appearance has profound implications for the identity hypothesis. The perceived bounded rectangle is a product of interpolation across two gaps (given by the two white vertical columns). This rectangle's orientation is a result of depth spreading. Stereoscopic disparity in this display is given only by the positions of the vertical edges of the central rectangle. The middle section of the display (two black squares separated by a white gap) is identical in the left eye's and the right eye's images. No information in the middle of the display indicates that the central square is in front of or behind either column. Thus, the slant of the rectangle is obtained via depth spreading from the disparity of the rectangle's vertical edges at its left and right ends. Depth spreading occurs within objects or continuous surfaces (Hakkinen et al., 1998); it does not spread unconstrained across a whole scene. Therefore, to be affected by depth spreading, the rectangle must be a unified object.

Figure 14. Display in which interpolation necessarily precedes determination of modal or amodal appearance of contours of surfaces. From "Segmentation and Grouping in Object Perception: A Four-Dimensional Approach," by P. J. Kellman, 2003, in *Perceptual Organization in Vision: Behavioral and Neural Perspectives* (M. Behrmann, R. Kimchi, & C. R. Olson, Eds.), p. 174. Copyright 2003 by Lawrence Erlbaum Associates. Reprinted with permission.

With this in mind, consider the modal or amodal appearances of various contours. Why is one vertical column modally completed and the other amodally completed? These effects are determined by the rectangle's slant in depth; depth spreading confers depth values all along its boundaries (including the interpolated parts). These (observer-relative) depth values are smaller than that of the left vertical column; thus, the rectangle here has illusory contours and appears in front. The left vertical column, therefore, has occluded contours where these surfaces cross. The situation is reversed on the right, where the rectangle passes behind the white column. Here, the rectangle's contours are occluded, and the right column's contours are illusory. The modal–amodal appearances of both the rectangle's contours and the white columns are consequences of parts of the rectangle having particular positions in space, given by depth spreading.

By now, the argument relevant to the identity hypothesis may be obvious: The amodal or modal appearances of several contours in this display are consequences of depth spreading. Depth spreading presupposes that the rectangle is a unified object. But the existence of the rectangle as a unified object is a result of contour interpolation. This bounded object does not exist in the stimulus! Thus, in this type of display, contour interpolation necessarily precedes determination of the appearance of contours as illusory or occluded. To review the logic, (a) interpolation produces the bounded rectangle. (b) Because the rectangle is a unified, bounded object, depth spreads along the rectangle from its endpoints to contour and surface points, giving it 3-D slant and depth positions all along its horizontal boundaries. (c) It is the relation of depth positions along the rectangle's contour to depth positions of the white columns that causes the rectangle to be seen as passing in front of one column and behind another. (d) This passing behind or in front of the columns' interpolated contours is what produces the amodal appearance of the left column and the modal appearance of the right column. (In complementary fashion, the modal and amodal appearances of the left and right parts of the rectangle's horizontal edges are also determined.) In short, modal or amodal appearance in this type of display presupposes interpolation. Interpolation necessarily precedes determination of modal or amodal appearance. Therefore, when interpolation first occurs, it cannot be amodal or modal; these labels designate the appearance of interpolated contours in the final scene representation.

Depth spreading and 3-D interpolation. Phenomena related to the depth-spreading display in Figure 14 provide a strong argument that the identity hypothesis applies to 3-D interpolation, not just to displays having frontoparallel (2-D) components. Figure 13C provides an example of a 3-D SSO in which one object penetrates another. Closely related displays are shown in Figure

15. Whereas in Figure 14, interpolation of the two vertical columns occurred in a frontoparallel plane and, therefore, involved 2-D interpolation, Figure 15 incorporates two crossing objects, both of which involve 3-D interpolation. In Figure 15A, a left-tilting bar appears behind the right-tilting bar. The perceived arrangement is opposite in Figure 15B. These arrangements appear to be unambiguous and stable. What determines them? The only differences between Figures 15A and 15B are small changes in disparity given to the physically specified edges of the bars. Note that because of the white circular region in the middle, there is no point at which physically specified edges of the two bars actually intersect. Therefore, the region in which the two bars cross is one in which there is no relative disparity information and no contour information at all in the stimulus patterns themselves. Both bars, however, undergo completion as unified objects with continuous contours as a result of 3-D contour interpolation. As a consequence of object formation, depth specified in the physically given contour regions spreads across interpolated regions, giving a perceptually determinate depth to all points along the objects' edges. As a result of this depth spreading, the object with the nearer depth coordinates is perceived as in front, and its interpolated boundaries acquire a modal appearance (i.e., there is no intervening opaque surface between the observer and these boundaries). The other object's interpolated boundaries take on an amodal appearance in the region where they pass behind the first object. Modal versus amodal appearance marks whether the interpolated parts of objects are in front of or behind other objects (Kellman, 2003a). As this example indicates, modal and amodal appearances do not designate separate interpolation processes, because the relative positions of the two objects are a consequence of interpolation. The same

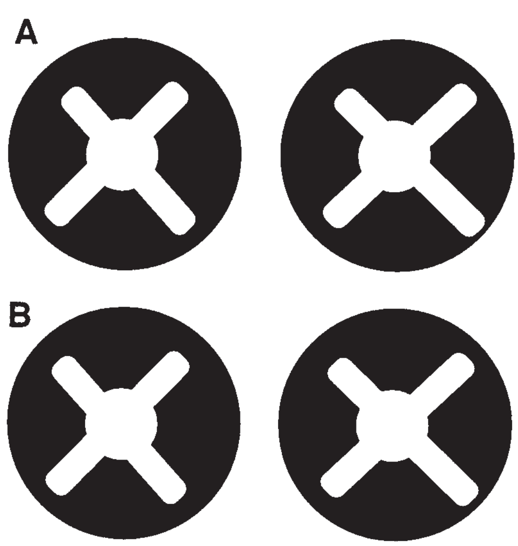

Figure 15. 3-D interpolation displays in which depth order depends on depth spreading. The displays are stereo pairs that can be free-fused by crossing the eyes. A: When cross-fused, the bar extending from the lower left to the upper right appears in front. B: Small changes in disparity of the physically specified edges of the bars causes the bar extending from bottom right to top left to appear in front. In both displays, depth order depends on relative depth values given by depth spreading from disparity specified at physically defined edges. Because depth spreading is confined within objects, interpolation necessarily precedes modal or amodal appearance.

explanation applies to the 3-D SSO in Figure 13C. The ribbon-like object has interpolated contours that appear illusory up to the point at which it passes through, and behind, the other object; the rest of its interpolated contour is occluded. But this stable impression presupposes that relative depths of the two objects are determined at each contour and surface point. Yet the contours in the crossing–penetrating area are interpolated, and their depth is received via depth spreading. The final modal–amodal appearance of contours and surfaces presupposes interpolation.

Against this background, how might one assess findings that have led some investigators to suggest that different processes or mechanisms are at work in producing illusory and occluded contours? Given a common interpolation step and the presence of other factors in determining a final scene representation (e.g., the depth relations in modal and amodal displays), one would expect some aspects of processing and neural responses to be in common and others to differ. Reported divergences are consistent with differences in processing related to differences in initially registering the different stimuli used for modal and amodal displays (e.g., Ringach & Shapley, 1996) or differences in constraints on final scene representations in different displays (see below). These differences are real and important, but they do not conflict with the notion of a common contour-interpolation process. Given the logical requirements of the phenomena described above—quasimodal interpolation, Petter-effect displays, and depth-spreading displays in which depth positions presuppose object formation—the notion of separate interpolation processes for modal and amodal configurations cannot be sustained.

The Promiscuous-Interpolation Hypothesis

An account that is consistent—both with a common interpolation process and the difference in appearance of final representations in modal and amodal cases—is what we have called the *promiscuous-interpolation hypothesis* (Guttman & Kellman, 2004; Kellman et al., 2001). On this hypothesis, contour interpolation happens among all relatable edges that lead into tangent discontinuities. Some of these connections made by interpolation processes will not survive into final scene representations. The reason is that a number of constraints operate in subsequent processing to determine consistent and coherent object representations in ordinary scenes. Such constraints might include edge labeling in terms of boundary assignment; depth relations in scenes; and, possibly, a constraint that a given edge can be connected by interpolation to only one other edge in the final scene representation. The promiscuous-interpolation hypothesis solves several problems in theories of object formation. First, it allows interpolation to occur without bringing to bear a great deal of higher level information, such as which way boundaries bound and whether connecting to edges will lead to a sensible object (or aperture) that can actually exist in space. Although models of object formation often require consistency of boundary assignment for interpolation to occur at all (e.g., Heitger et al., 1998), this constraint is not supported empirically. As Figure 16 shows, for example, interpolation may occur despite conflicting boundary assignment. In the figure, alternating inducing elements have opposite boundary assignments, in all cases given by perhaps the most powerful indicator of boundary assignment and relative depth, stereoscopic disparity. Nevertheless, a complete, circular boundary (perhaps belonging to

Figure 16. An example of robust interpolation despite conflicting boundary assignment. The display is a stereo pair that can be free-fused by crossing the eyes. Alternating physically defined edges around the circle have opposite boundary assignments given by stereoscopic depth information. Nevertheless, clear contour interpolation is observed.

some glass object) is seen. Second, the promiscuous-interpolation process is consistent with data on the microgenesis of interpolated contours. Estimates of around 120 ms for interpolation have emerged from studies in different paradigms (Guttman & Kellman, 2004; Guttman et al., 2003; Ringach & Shapley, 1996). If the visual system applied more elaborate scene constraints to determine whether interpolation should occur in the first place, substantially longer latencies might be expected. Third, the promiscuous-interpolation hypothesis explains experimental data on interpolation that are difficult to account for otherwise (Guttman & Kellman, 2005). For example, it has been known for some time that illusory-contour displays in which the inducing elements are presented as outline forms give very weak or nonexistent illusory contours (e.g., Kellman & Shipley, 1991). This fact is quite troublesome for neural-style models of interpolation that use edge detectors as inputs, especially edge detectors that sum across odd- and even-symmetric cells (which respond best to surface edges and thin lines, respectively; e.g., Heitger et al., 1992; Morrone & Burr, 1988). Guttman and Kellman (2004) found that outline figures and other displays that have relatable edges but produce weakly perceived illusory contours actually produce comparable effects to optimal illusory and occluded contours in objective paradigms assessing contour interpolation (e.g., the fat–thin task of Ringach & Shapley, 1996). The theoretical proposal here is that early interpolation is relatively unconstrained; however, a number of connections made by interpolation processes will not appear in the final scene representation. These connections may nevertheless affect performance on tasks sensitive to the early interpolation of edges.

For the present purposes, the strength of the promiscuous-interpolation hypothesis is that it fits with three phenomena that make a logical case for the identity hypothesis: quasimodal interpolation, Petter-effect displays, and the depth-spreading displays described above. In many cases, at the time that interpolation occurs, aspects of the final scene arrangement (e.g., which parts of a contour go in front of or behind other surfaces) may not yet be worked out. Processes that impose constraints on scene coherence and consistency work with the results of interpolation to produce amodal and modal appearances of contours. In some cases, these scene constraints may actually bar some interpolated contours from being realized in the final representation, as in the case of transparency analyzed by Singh and Anderson (2002).

In short, the argument for a common interpolation process based on cases in which interpolation precedes determination of modal versus amodal appearance is closely tied to 3-D interpolation. The identity hypothesis in contour interpolation is best thought of as characterizing interpolation in all three spatial dimensions.

Contour and Surface Processes in 3-D Interpolation

There are similarities between the geometry of contour relatability and some computational vision research regarding surface interpolation from points or patches along a surface given by stereoscopic depth information. Grimson (1981) proposed an elegant mathematical theory indicating how 3-D surfaces may be constructed from spatially separated visible features. Zero-crossings in luminance distributions (e.g., Marr & Hildreth, 1980) provide the input to a stereo-matching algorithm (Marr & Poggio, 1979), leading to observer-relative depth values at a number of visible features. Perception of a surface is accomplished by construction of a smooth surface that encompasses the visible features. The form of this surface is given using the calculus of variations to minimize quadratic variation, a functional of the following form:

$$\theta(f) = (\int\int f_{xx}^2 + f_{xy}^2 + f_{yy}^2)dx\, dy)^{1/2}.$$

Although all visible points are initially fit by a smooth surface, Grimson (1981) also proposed a method of finding discontinuities in surfaces. This is done, in a manner analogous to edge finding in luminance distributions, by using zero-crossings in the second derivative of depth values. If, after the initial surface fit, a zero-crossing is detected between visible surface features, the algorithm assigns a surface discontinuity there.

Grimson's (1981) theory does not directly address either amodal or modal completion. Moreover, current views of stereoscopic surface perception suggest that the approach of reconstructing a surface from observer-relative depth values of a number of features is incorrect (Howard & Rogers, 1995). Nevertheless, the mathematics of surface smoothness and discontinuity in Grimson are related to the present proposals regarding contour relationships. They differ in that Grimson's smoothness criteria are not based on contour relations, and they involve neither the initiating condition of tangent discontinuities nor any 90° constraint. Nevertheless, given the similarities regarding smoothness, an interesting question is whether a more unified treatment of surface reconstruction and contour interpolation may be possible.

Distinct but complementary processes of contour and surface completion in object formation have been suggested by a number of investigators (e.g., Grossberg & Mingolla, 1985; Kellman & Shipley, 1991; Kellman et al., 1998). In 3-D-interpolation experiments, there have also been indications that contours are special. Specifically, in one of their experiments, Kellman et al. (2005) found that misaligning contours laterally removed the performance advantage for otherwise 3-D-relatable displays. From the standpoint of surface interpolation in Grimson's (1981) algorithm, the surface points of each tab would obtain depth values by interpolation from the bounding contours (given by disparity). Then, the surface values within the tabs could be connected by surface spreading across the gap. Such a display can be seen in Figure 17D. Indeed, inspection of the display indicates some perceived connection of the surfaces. However, because of the nonrelatability of the bounding contours, no clearly defined boundaries of the object are seen, and experimentally, the effects of object formation disappeared. Likewise, the relatable tabs with rounded corners,

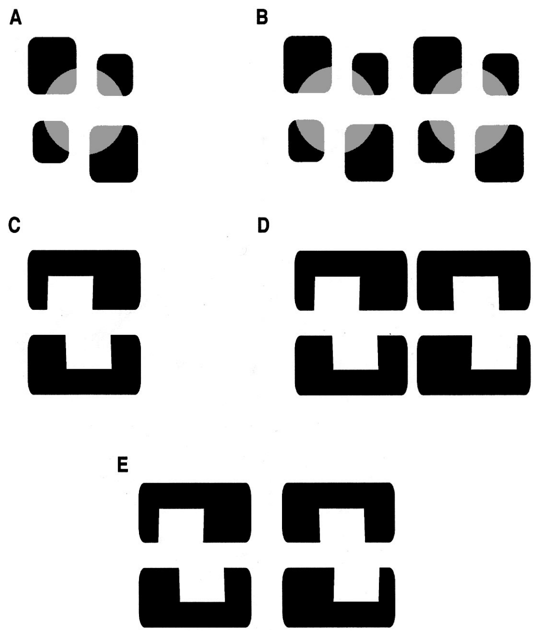

Figure 17. Relations between surface spreading and depth separation. A: Ambiguous transparency: The central object formed by interpolation may be perceived as transparent and in front of other surfaces or as partly occluded and behind other surfaces. B: Unambiguous transparency: The display is a stereo pair that can be free-fused by crossing the eyes. C: The display lacks relatable contours and shows minimal surface spreading. D: The display lacks relatable contours, but separation in depth from background shows obvious surface spreading. The display is a stereo pair that can be free-fused by crossing the eyes. E: Illustration of confinement of surface spreading within real and interpolated contours and their linear extensions. Unlike the surface spreading observed in Panel D, a shift in depth of the lower white tab in this display appears to block any perceived connection between top and bottom tabs on the basis of surface spreading. From "3-D Interpolation in Object Perception: Evidence From an Objective Performance Paradigm," by P. J. Kellman, P. Garrigan, T. F. Shipley, C. Yin, and L. Machado, 2005, *Journal of Experimental Psychology: Human Perception and Performance, 31,* p. 571. Copyright 2005 by the American Psychological Association. Adapted with permission.

tested in Experiment 4 of Kellman et al. (2005), would be readily incorporated into a single surface in the Grimson scheme, yet the experimental result was that the absence of tangent discontinuities eliminated any perceptual connection between these visible regions separated by a gap.

It appears that relations of surface patches alone do not govern object formation or even surface spreading. What governs surface spreading in 3-D interpolation? In two dimensions, it has been theorized that surface spreading occurs up to real and interpolated edges and also along the linear extensions of occluded edges that end in tangent discontinuities but do not connect to others (Kellman & Shipley, 1991). Experimental evidence has supported these claims (Kellman et al., 1998; Yin et al., 1997). This interpretation appears to apply to 3-D interpolation as well. In the display in Figure 17D, 3-D contour interpolation cannot occur because edges have been misaligned laterally to disrupt 3-D contour relatability. Yet surface spreading within the linear extensions of the two tabs allows a surface connection (with uncertain boundaries) to form between the top and bottom tabs. Introducing 3-D information may actually enhance surface spreading, as can be seen by comparing Figures 17D and 17C and also Figures 17B and 17A (for the case of spreading that leads to perceived transparency). The reason is that in 3-D, interpolated surfaces may occupy depth positions clearly different from the background, and they also can curve in depth.

The relevant linear extensions appear to be vectors in 3-D space. As Figure 17E shows, shifting one surface in depth relative to the other destroys the impression of connection between the surfaces.

In sum, our theory of 3-D object formation posits that the 3-D geometry of contours is complemented by 3-D surface-spreading processes. This view incorporates the important role of contours as well as prior theoretical notions about surface relationships (e.g., Grimson, 1981) and empirical results of surface interpolation in structure-from-motion displays (Saidpour et al., 1994).

Although the evidence suggests that contour processes are crucial in forming and segmenting bounded objects, it is nevertheless striking that the geometry of 3-D contour interpolation has much in common with some surface-interpolation algorithms. One way to subsume surface and contour processes in a common framework would be if the visual system somehow derived orientation vectors (orientations of tangent lines) from surface patches having no visible edges. These orientation vectors could accomplish interpolation for surface patches in the same manner as we have proposed for 3-D contour interpolation.

There are a number of issues that would have to be resolved in such a scheme. Moreover, given the evidence that contour and surface processes exert different effects in object-formation experiments, the best tentative conclusion may be that 3-D contour and surface interpolation are constrained by similar geometries but are carried out by complementary processes.

Global and Local Processes in Object Perception

The account of 3-D object formation based on 3-D relatability encompasses relatively local determinants of object perception. It has often been argued that in addition to these, there are more global influences, such as object symmetry, regularity, or familiarity. In this section, we consider the relation of the relatively local account that we have described to work suggesting global influences. Much of our discussion involves 2-D phenomena, because these have dominated research on the question of global influences. However, the issues apply to both 2-D and 3-D interpolation.

A seminal proposal regarding global influences on object formation was the Gestalt principle of *Prägnanz*, the idea that perceptual processes tend toward simple or regular (symmetric) outcomes (e.g., Koffka, 1935). Later work attempted to put this notion on a more formal footing (e.g., Buffart, Leeuwenberg, & Restle, 1981; Hochberg & McAllister, 1953), and some experimental results suggest a role of symmetry in object completion (Sekuler et al., 1994; van Lier, van der Helm, & Leeuwenberg, 1994; for a recent review, see van Lier, 2001).

Both the role of global factors and the relationship between global and local factors in object formation remain unsettled. Kanizsa (1979) argued that neither global symmetry nor knowledge about objects has much impact on the basis of displays that pitted these factors against local edge continuity. Likewise, Michotte et al. (1964) presented evidence that specific knowledge of an occluded object, even over large numbers of learning trials, did not affect completion that followed local edge continuity.

Evidence from priming studies, however, has tended to show global effects along with local ones (Sekuler et al., 1994; van Lier et al., 1994). van Lier, van der Helm, and Leeuwenberg (1995) argued from their results for dual or multiple representations activated by partly occluded displays. Figure 18A shows an example similar to the displays used by Sekuler et al. (1994). (Either view in the stereo pair is similar to the displays used by Sekuler et al., 1994, which did not include stereoscopic depth information.) Priming results suggested that in partly occluded displays of this sort, subjects were completing a fourth articulated part behind the occluder. One interpretation of this result is that interpolation of contours can be influenced by detection of global symmetry information.

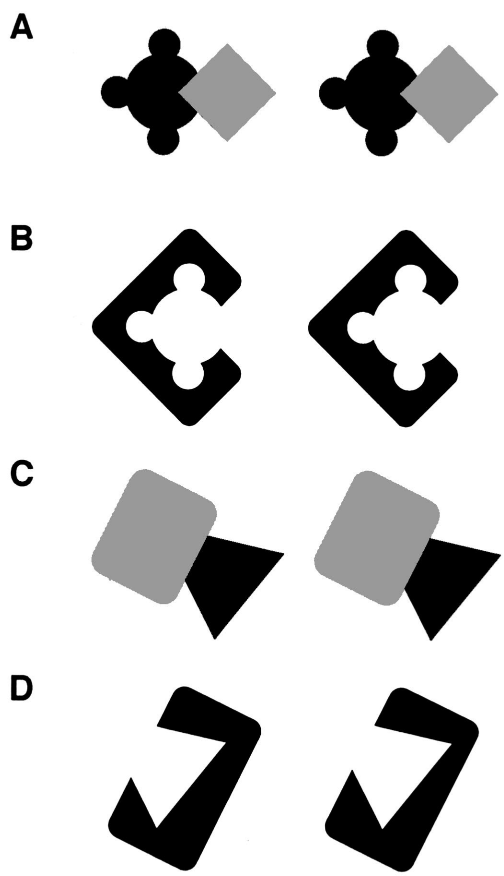

Figure 18. Separating processes in object perception. All of the displays are stereo pairs that can be free-fused by crossing the eyes. Stereoscopic viewing enhances the effects but is not necessary, as can be noted by viewing a single image of each pair. A: Example of occlusion display used to study global influences on completion. B: Illusory-object version of the occlusion display in Panel A; no fourth articulated part is seen. C: Occlusion display in which observers may report that the partly specified figure is a triangle. D: Illusory-object version of the occlusion display in Panel C; no clear third vertex of the triangle is seen. According to the hypothesis of recognition from partial information, contour-interpolation processes do not produce globally symmetric completions in any of these displays. In the occlusion displays, observers perceive that parts of the figures are out of sight, allowing the possibility of reports that the objects continue symmetrically behind the occluders.

Does the visual system actually interpolate edges to construct symmetric object representations? Do global processes interact with local ones in specifying interpolated contours and surfaces? There are indications that global recognition and local interpolation processes are distinct and operate at different levels. One indication comes from the identity hypothesis, which posits a common interpolation step in the formation of occluded and illusory contours. Figure 18B shows an illusory-contour display that has physically given edges equivalent to the occlusion display in Figure 18A. If global completion produces symmetric perceived objects, the four-lobed figure should be apparent, complete with clear illusory contours, in Figure 18B. In fact, however, no such effect is evident in Figure 18B. A similar contrast can be seen in Figures 18C and 18D. Although one can easily recognize that the third vertex of a symmetric (isosceles) triangle may lie behind the occluder in Figure 18C, the equivalent illusory-contour case in Figure 18D reveals an indeterminate continuation of the triangle's physically given edges because of their nonrelatability. To our knowledge, there are no reports of completion on the basis of global symmetry, simplicity, or familiarity among the hundreds of published articles on illusory contours. Contour interpolation on the basis of relatability operates for both illusory and occluded objects, but global processes apparently do not.

Kellman et al. (2001) pointed out that there are different ways that global information might play a role in object perception. Relatively local interpolation processes may specify precise boundary positions, but global completion effects may not. Global effects may be explained by what Kellman et al. (2001) termed *recognition from partial information* (RPI). RPI occurs when the products of some fairly early stage of processing activate a later representation, which can then be used in some cognitive task. However, the activation of the late representation does not influence an earlier processing component in any way. Specifically, although an occlusion display may activate a memory representation of a symmetric or familiar object, this activation does not in turn lead to specification of local interpolated boundaries. Kellman (2003b) gives the following example of RPI:

> If you see the tail of your calico cat protruding from under the sofa, you may easily recognize and report that the cat is present, even though the particular contours and surfaces of the hidden parts of the cat are not given perceptually. A stored representation of the cat may be activated and a belief about its presence may be formed. But RPI differs from perceptual processes that actually specify the positions of boundaries and surfaces behind an occluder. (p. 195)

Figure 19 contrasts the two processes. In Figure 19A, RPI operates. If one is asked what one "sees" behind the occluder, the answer would be a cat; the head and tail appear to be connected. Even in Figure 19B, with only the tail visible, one might believe there is a cat behind the occluder (especially if one had previously seen a cat with such a tail). Figure 19C introduces contour relatability, producing the odd percept of a cat connected to a vase. The cat and its tail are not connected by contour-interpolation processes, and each is connected by these processes to something else (despite the affront to common sense and certain Bayesian priors).

In Figures 19A and 19B, a stored object representation is activated despite the lack of specific information about the location of

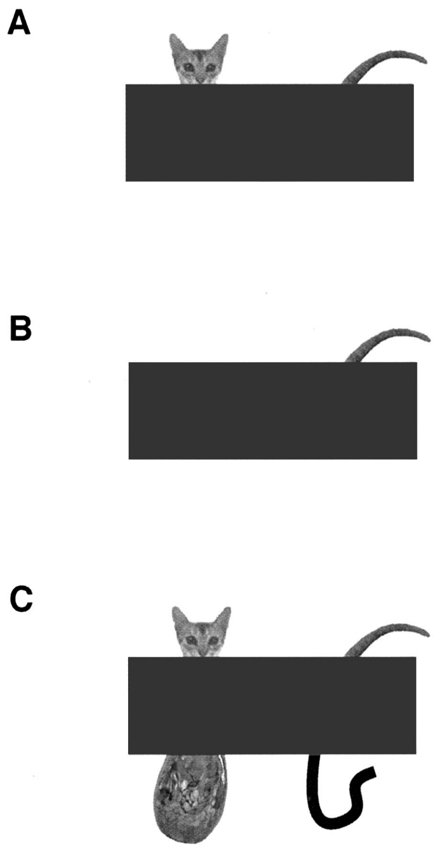

Figure 19. Examples of recognition from partial information and contour-interpolation processes. A: Observers may report a cat behind the occluder and believe that the head and tail are connected. B: Even the tail alone may be sufficient to evoke reports or beliefs about a cat behind the occluder. C: Contour-interpolation processes connect the cat's head to the vase and its tail to the curved line; they do not connect the cat's head and tail, although an observer may still believe in or report such a connection.

local contours. This activation does not feed back and specify, via contour interpolation, the complete contours of the cat; their exact positions remain indeterminate. Yet priming effects may nevertheless occur as a result of the activation of the familiar (or, in other cases, of a symmetric) object representation. The fact that data supporting global effects in completion come solely from priming and perceptual-report paradigms is consistent with this possibility. It is known, for example, that priming is susceptible to influences at many levels, including high-level cognitive as well as basic perceptual ones (e.g., Kawaguchi, 1988). Priming studies showing global completion effects have typically used large numbers of trials with a small set of symmetric and/or familiar figures, and the design of such studies requires that fully symmetric, unoccluded displays be shown on many trials. Even if subjects start out with little familiarity or do not notice the possibility of symmetry under occlusion, repeated exposure may produce familiarity or symmetry responses. To our knowledge, no study has disentangled the effects of global representations formed across trials from those that might occur on initial viewing of a particular occluded stimulus. Of course, in many situations, both local boundary interpolation and RPI may proceed in parallel. This idea is consistent with the suggestion that multiple representations may emerge from processing (Parovel & Vezzani, 2002; van Lier et al., 1994).

Why might RPI operate only in occluded displays? The answer is that despite a shared local interpolation step, illusory and occluded objects do have important differences in the final scene representation. Specifically, illusory objects are seen as nearest to the observer in some visible direction, whereas occluded objects are not. In other words, both amodal and modal contours and surfaces are represented, but whether a surface is behind another, relative to the observer's position, is marked in perceptual representations (Kellman, 2003a). The difference is important: A surface nearest in some visible direction may be contacted directly by reaching, whereas a surface behind some other surface may not. The visual system appears to represent the occluded parts of objects such that the observer is aware that part of the object is hidden from view, behind some other surface(s). This fact of occlusion may allow certain kinds of reasoning and responses that are not sensible when no part of an object is occluded. In particular, despite any local completion process, the observer can notice that unoccluded parts are consistent with some symmetric or familiar object. In Figures 19A or 19B, for example, recognizable parts of a cat may be sufficient to activate some representation of a whole cat.

Contour interpolation effects based on relatability and global effects in occlusion may indicate the operation of two distinct processes producing different kinds of effects (Kellman, 2001). The former is a bottom-up, relatively local process that produces relatively precise representations of boundaries. This process is perceptual in that it involves a modular process that takes stimulus relationships as inputs and produces boundaries and forms as outputs. The other process, RPI, is more top-down, global, and cognitive, coming into play when familiar or symmetric forms can be recognized.

Can RPI and local interpolation effects be distinguished experimentally? RPI may allow global recognition and priming effects, but it may not produce precise local, interpolated edges. Using a dot-localization paradigm, Kellman and colleagues (Guttman & Kellman, 2004; Guttman et al., 2003; Kellman, Temesvary, Palmer, & Shipley, 2000) found evidence that perceptual boundary-completion processes lead to highly precise representations of boundary locations in both occluded and illusory displays. Although details of the paradigm may be found elsewhere (Guttman & Kellman, 2004; Guttman et al., 2003; Kellman et al., 2000), the gist can be easily conveyed here. Precision of boundary location can be measured by showing an occluded or illusory display and briefly flashing a probe dot. Subjects were instructed to respond on each trial as to whether the probe dot fell inside or outside the object's boundaries. Using adaptive staircases to obtain two points on the psychometric function (probability of seeing the dot inside vs. outside the object), the researchers were able to estimate both the location of interpolated boundaries and the precision with which they were represented.

Kellman and colleagues (Guttman & Kellman, 2004; Guttman et al., 2003; Kellman et al., 2000) predicted that examples of so-called global completion would not give rise to precise local boundaries, whereas interpolation based on contour relatability would. These are exactly the results that were obtained (Guttman & Kellman, 2004; Kellman et al., 2000). Localization of boundaries in displays in which completion is predicted by relatability is extremely accurate and precise, but for cases of global symmetry, it is both imprecise and inaccurate, often worse than in the local case by almost an order of magnitude. These outcomes are consistent with the idea of separate perceptual-completion and more cognitive RPI processes.

This notion of separable processes may help in understanding each type of process individually and in understanding their relationship better. For example, what would be required for the visual system to use notions of object symmetry or regularity in perception of ordinary scenes? The examples of symmetry influencing completion in the research literature are typically singly occluded and composed of geometric primitives, as in Figure 20A. For symmetry to be useful, however, it must be available under more ecologically valid conditions. But symmetry is a complicated notion, and it requires information about global structure that generally is not available until basic segmentation and grouping processes, such as local contour interpolation, have formed a completed object (or at least substantial parts of one; see Figure 20B). It is unlikely that disparate parts of an object viewed under partial occlusion will have any type of symmetry. In complex scenes, obtaining reasonable inputs to a symmetry processor would seem to presuppose local interpolation processes that specify the connections of visible areas.

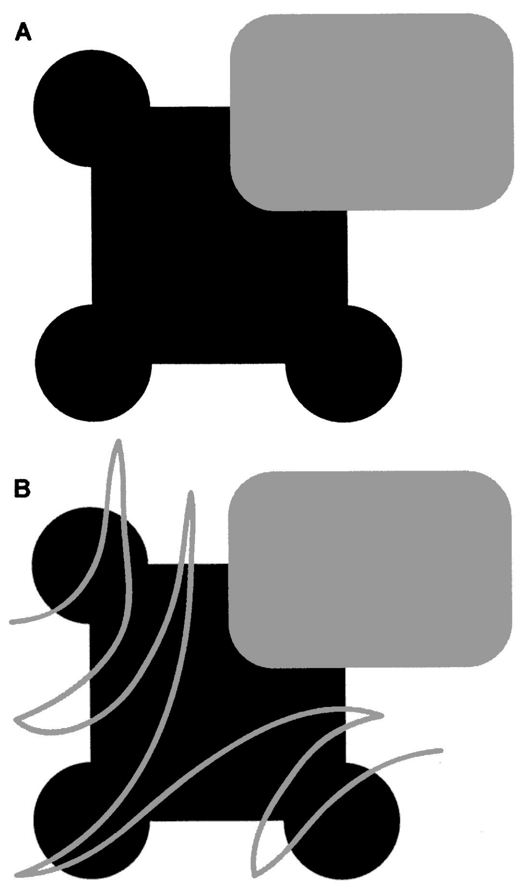

Figure 20. Example of difficulties in a symmetry-based interpolation scheme. A: Claims for symmetric completions of partly occluded objects usually use examples of singly occluded objects, in which aspects of symmetry or regularity may be extracted from single visible regions. B: In the more general case in ordinary scenes, objects may have complex patterns of occlusion. Extraction of symmetry or regularity presupposes connection of retinally separated visible areas by means of more local interpolation processes.

Neural Models, Computational Models, and 3-D Interpolation

The theory presented here offers a formal specification of the geometry of contour interpolation in three dimensions. The requirements of that geometry, and the available empirical evidence (e.g., Kellman et al., 2005), are unequivocal in implicating the 3-D positions and orientations of visible edges as inputs to the process of interpolation. These results seem likely to require major changes in computational and neural models of object perception processes. Most current models of edge and junction detection (e.g., Heitger et al., 1992; Morrone & Burr, 1988), contour integration (Field et al., 1993; Yen & Finkel, 1998), and contour interpolation (Fantoni & Gerbino, 2003; Grossberg, Mingolla, & Ross, 1997; Heitger et al., 1998) take as their inputs oriented units in early visual cortical areas. These units, as ordinarily interpreted, signal orientation as it exists on the retina. In other words, their variation in terms of orientation is captured by the possible orientations that a vertical edge could assume if it rotated around the line of sight. In short, the major processes leading to the perception of objects, from edge detection to unit formation, have most often been studied and modeled within the frontoparallel plane. Because it has been considered an advantage in linking perceptual phenomena to the functions of known neural units, models usually take as their inputs simulated activations of units in early cortical areas (e.g., the *S* and *C* operators in Heitger et al., 1992, which correspond closely to simple and complex cells found in V1 and V2).

The 2-D orientation characteristics of cells in these early visual areas are not sufficient to provide the basis of 3-D interpolation. One might suppose that outputs from these cells in combination with disparity information could do so. However, there are a number of obstacles for any simple use of orientation-sensitive units plus disparity. First, evidence suggests that the kind of disparity information available in these early areas is not likely the direct basis of the perception of depth intervals in the world (Cumming & Parker, 1999). Whereas relative disparities between points are needed for depth computations, those V1 neurons that show disparity sensitivity seem to be attuned to absolute retinal disparities. The latter vary with fixation. For example, in looking at a particular pair of points at different depths, an observer can make the absolute disparity of either point zero by fixating that point. Relative disparities, however, remain invariant with fixation changes. Cumming and Parker (1999) used a display with both kinds of disparity and found that when absolute disparities were added without changing relative disparities, responses from neurons were predicted by absolute but not relative disparities.

Second, relative disparities do not themselves allow perception of depth intervals in the world. Disparity information must be combined with egocentric distance to at least one point (Wallach & Zuckerman, 1963). Figure 21A shows an example in which two points, *X* and *Y*, are separated by a depth interval *d*. The problem of stereoscopic depth constancy is that the relative disparities (or disparity difference) between *X* and *Y* will decrease roughly as the square of viewing distance (Wallach & Zuckerman, 1963). A

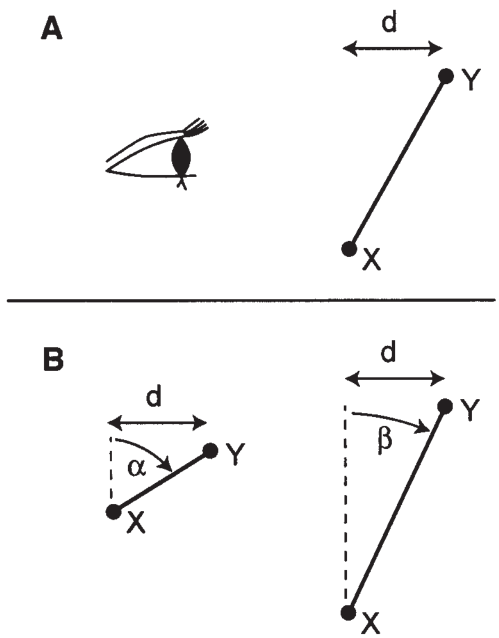

Figure 21. Relations between disparity, edge length, and slant. A: A given depth interval *d* in the world will give rise to decreasing disparity differences between Points *X* and *Y* as viewing distance increases. B: Slant depends on both the depth interval between two points and their separation. The two cases shown have the same depth interval *d* between Points *X* and *Y*; however, because the separation of *X* and *Y* differs in the two cases, Slant $\alpha >$ Slant β. From "3-D Interpolation in Object Perception: Evidence From an Objective Performance Paradigm," by P. J. Kellman, P. Garrigan, T. F. Shipley, C. Yin, and L. Machado, 2005, *Journal of Experimental Psychology: Human Perception and Performance, 31,* p. 581. Copyright 2005 by the American Psychological Association. Reprinted with permission.

further problem concerns slant perception. Perception of 3-D slant depends not merely on the depth interval between two points but also on their spatial separation. This problem is illustrated in Figure 21B. The two displays have the same depth interval between Points *X* and *Y* but different slants. Our experimental data suggest that edge segments at particular slants provide the inputs to interpolation processes. If this is so, where might neural units that encode this kind of information be found? No existing evidence suggests that it can be found in the earliest cortical visual areas, V1 and V2, although their outputs may well form the basis of subsequent computations (and they may also receive recurrent activation or inhibition from computations at higher levels).

Earlier, we mentioned a possible neural basis for the inputs needed for 3-D interpolation. Sakata et al. (1997) found evidence that slant is encoded by units in cIPS. Given that responses of these cells were triggered either by stereoscopic or texture information specifying the same slant, they would appear to be encoding exactly the kind of information needed to provide the substrate for 3-D interpolation. Moreover, the kinds of interactions among oriented units envisioned in some models (e.g., the *association field* of Field et al., 1993) could be carried out in a network of these units. Our data are consistent with a geometry of 3-D relatability governing the formation of objects across gaps in 3-D space. Interactions in a network of units encoding 3-D position and orientation would appear to be needed to accomplish 3-D object formation, and these recent neurophysiological findings suggest a plausible neural basis for such computations.

Traditionally, most topics in vision science have been studied, at least initially, using analyses of information in 2-D images. There are many good reasons for this tendency, including simplicity, the retinotopic mapping in early cortical areas, the kinds of oriented units found in those cortical areas, and so on. When 3-D considerations arise, there are several possible ways to incorporate them. One is to preserve an essentially 2-D approach and to add in some relatively external modifications or exceptions. Another is to generalize processes from a single frontoparallel plane to several or many such planes. Both of these approaches have the advantage of retaining 2-D operations and allowing direct links to known or likely characteristics of neural mechanisms in early cortical areas. A third approach is more radical. Processes and representations may rely on 3-D information in ways that cannot be realized in a 2-D framework with corrections.

The current theory of 3-D relatability and its supporting data and arguments imply that the third, radical approach is likely to be correct. Early demonstrations indicating some 3-D influences on interpolation could not readily decide this question. A theorem of projective geometry (Gans, 1969) states that a number of important properties in 3-D space—such as collinearity, smooth curvature, and sharp corners—are preserved when 3-D structures are projected onto 2-D images (barring degenerate cases, as when a line projects to a point). One way of dealing with basic phenomena showing that interpolation is a 3-D process is to use this projective invariant to allow a shortcut. 3-D interpolation configurations could be processed via their 2-D projections to determine what gets connected to what, and then the apparent depth could be added (e.g., by using information from disparity-sensitive neurons), perhaps triggered by certain features indicating depth or more general object-recognition processes. An even more sophisticated interpolation scheme would allow completion of objects to occur at different depths but would use multiple frontoparallel layers, a useful technique in some computational vision applications (Adelson, 1995). In such a scheme, models indicating how interpolation occurs in a frontoparallel plane could still be applied.

The theory we have presented here, and the empirical support we have noted for it, make clear in a way previously not decidable that object formation must be a truly 3-D process in the more radical sense. This is the case for at least two reasons. First, the theory specifies, and the data confirm, that misalignments in depth that do not exceed the tolerance for 2-D misalignment are sufficient to disrupt 3-D object formation. Note that projective geometry guarantees that 3-D collinearity will project to 2-D collinearity, but not the converse. In other words, not all 2-D-relatable edges are 3-D relatable. Our theory indicates that edges will be perceptually disconnected if they fail to meet certain 3-D geometric criteria. Finally, the idea of multiple frontoparallel layers will not work in this domain. The current account of 3-D object formation implies that the visual system represents and utilizes positions and orientations of edges in 3-D space (i.e., edges not lying in any frontoparallel plane) to determine whether an object is formed across a gap or not. We have explained above why this characterization will require a fundamentally different character for neural models of interpolation. Understanding the ways in which perception is truly dependent on 3-D relationships is a high priority throughout the field. The current theory of 3-D interpolation, in suggesting that 2-D representations plus corrections cannot account for 3-D aspects of object formation, may lead to more general and accurate accounts of object perception.

Conclusion

The present work casts object formation as a truly 3-D process and specifies much of the geometry governing that process. Interpolation processes take as inputs the positions and orientations of visible edges in 3-D space, and they produce as outputs representations of contours and surfaces spanning all three spatial dimensions.

We have described the geometric relations among visible edges that allow 3-D interpolation to occur in terms of the theory of 3-D relatability. Along with other determinants of interpolation, such as tangent discontinuities in visible contours, the theory captures the conditions under which 3-D interpolation does and does not occur. 3-D relatability answers a number of questions in the geometry of object formation, and it raises some wholly new ones, such as that of the role of torsion.

As we have discussed, placing object formation on a truly 3-D footing has many implications for fundamental issues in object perception generally. The theory clarifies the relations of contour and surface processes in three dimensions. Contour processes lead, and surface interpolation occurs within real contours, interpolated contours, and the linear extensions of occluded, noninterpolated contours. The analysis also helped to reveal and elaborate the 3-D nature of the identity hypothesis—an underlying unity beneath different-looking appearances of interpolation. We described the promiscuous-interpolation hypothesis, by which oriented units in a network of 3-D relations may accomplish initial connections of contours across gaps. These in turn are subjected to subsequent constraints in determining final scene representations. Explication of a theory of 3-D interpolation also helps to separate processes operating at different levels in object perception and recognition, such as relatively local contour-interpolation processes and sensitivity to global symmetry.

Some of the most far-reaching implications of 3-D relatability involve neural-style models of object formation. Unlike current models using 2-D orientations and relations, 3-D interpolation requires units that encode 3-D positions and orientations and mechanisms that implement the geometry of 3-D relatability in their interactions. These considerations suggest that object formation may involve information and cortical areas different from those previously believed. The issue of what is connected to what in the world, and of how one can determine the connected objects in the world through reflected light, is among the most important priorities for beings who see. It is equally high on the agenda of scientific efforts to understand vision. Characterizing 3-D interpolation in object formation and its operation according to the geometry of 3-D relatability may improve current understanding of visual phenomena, and it will hopefully spur further progress in understanding these complex and important processes.

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